Level 2

FURTHER MATHEMATICS

#MathsConf29: Progress your highest attainers towards A-level success with L2FM

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## Starter questions to try

The below are all taken from Exampro and have been selected to give you a flavour of some of the slightly trickier questions on the specification.

### Question 1

#### Question 18, Paper 1, June 2019

*ADEF* is a trapezium.

*ABCD* is a straight line.

*BCEF* is a square of side  cm



(a)  Show that *AB* =  cm

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**(1 mark)**

(b)  Show that *DE* = 2  cm

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**(1 mark)**

(c)  Work out the perimeter of the trapezium *ADEF*.

Give your answer in the form *t*  + *w*  where *t* and *w* are integers.

You **must** show your working.

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Answer \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ cm

**(3 marks)**

Question 2

Question 17, Specimen Paper 2

By multiplying both sides of the equation by *x*

Solve 2*x* − 3*x* = 7*x* − for *x* > 0

Give your answer to 3 significant figures.

You **must** show your working.

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Answer \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**(4 marks)**

Question 3

Question 12, Paper 1, June 2019

A curve has the equation *y* = *x*3 + *ax*2 − 7 where *a* is a constant.

The gradient of the curve when *x* = 4 is **twice** the gradient of the curve when *x* = −1

Work out the value of *a*.

You **must** show your working.

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Answer = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**(5 marks)**

Question 4

Question 11, Specimen Paper 1

A circle, centre *C*, touches the *y*-axis at the point (0, 2)

The line *y* = *k* intersects the circle at the points (1, *k*) and (5, *k*)



Work out the equation of the circle.

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Answer \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**(3 marks)**

Mark schemes

Question 1

Question 18, Paper 1, June 2019

|  |  |  |  |
| --- | --- | --- | --- |
|   | **Answer** | **Mark** | **Comments** |
| (a) | **Alternative method 1** |
|   |   | B1 | oe must see tan60 oe and some evidence of manipulation with  oe as well as the final answer to award B1 |
|   | **Alternative method 2** |
|   | Use of 1 : 2 :  triangle and showing that our triangle is an enlargement scale factor   | B1 | oe must see the triangle drawn and labelled or the ratio clearly seen **and** the scale factor clearly stated |

|  |  |
| --- | --- |
| (b) | **Alternative method 1** |
|   |   | B1 | oe must see sin30 oe and some evidence of manipulation with 0.5 oe as well as the final answer to award B1 |
|   | **Alternative method 2** |
|   | Use of 1 : 2 :  triangle and showing that our triangle is an enlargement scale factor   | B1 | oe must see the triangle drawn and labelled or the ratio clearly seen **and** the scale factor clearly stated |

|  |  |  |  |
| --- | --- | --- | --- |
| (c) |  or Text, letter  Description automatically generated or  so   | B1 | oeallow  or  for this mark seen on the diagram or clearly shown in working |
|   |  or *CD = DE* cos 30°   Text  Description automatically generated or   | B1 | oeallow  or  or   for this mark seen on the diagram or clearly shown in working |
|   |   | B1dep | dependent on B1, B1 already awarded |

|  |  |
| --- | --- |
|   | **Additional Guidance** |
|   | Condone brackets missed off if recovered |
|   | *AF* and *CD* could be seen in part (a) or part (b) so could be awarded B1 in part (c) if used correctly |

Question 2

Question 17, Specimen Paper 2

|  |  |  |  |
| --- | --- | --- | --- |
|   | **Answer** | **Mark** | **Comments** |
|   | 2*x*2 − 3*x* = 7 | M1 | at least two terms correct |
|   | 2*x*2 − 3*x* − 7 (= 0) | A1 | oe 3-term quadratic equation |
|   | Text, letter  Description automatically generated or   | M1 | oecorrect attempt to solve their 3-term quadratic equation |
|   | 2.77 | A1 | 2.77 and − 1.27 is A0 |

Question 3

Question 12, Paper 1, June 2019

|  |  |  |  |
| --- | --- | --- | --- |
|   | **Answer** | **Mark** | **Comments** |
|   | 3*x*2 + 2*ax* | M1 | allow a derivative with at least one term correct and a term in *a*eg 3*x*2 + 2*ax* + 7 or 3*x*2 + 2*a* |
|   | 3(4)2 + 2*a*(4) or 48 + 8*a* | M1 |   |
|   | 3(−1)2 + 2*a*(−1) or 3 − 2*a* | M1 |   |
|   | 48 + 8*a* = 2(3 − 2*a*) | M1dep | oe ft if first M1 earned |
|   | (*a* =) −3.5 | A1 | oe |

|  |  |
| --- | --- |
|   | **Additional Guidance** |
|   | Minimum expected working is to see the correct derivative in the first M mark. If no working seen then no marks can be awarded |   |
|   | If the word "twice" is interpreted the wrong way round ie equation becomes2(48 + 8*a*) = 3 − 2*a* this gives an answer of *a* = −5 or −5.1666... | M1, A1, A1, M0, A0 |

Question 4

Question 11, Specimen Paper 1

|  |  |  |  |
| --- | --- | --- | --- |
|   | **Answer** | **Mark** | **Comments** |
|   |  or 3or  or 3 | M1 | may be implied |
|   | (*y*-coordinate of *C* =) 2 | M1 | may be implied |
|   | (*x* − 3)2 + (*y* − 2)2 = 9 | A1 | allow (*x* − 3)2 + (*y* − 2)2 = 32 |

## AQA resources for Level 2 Further Mathematics

AQA Overview page

[aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/introduction](https://www.aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/introduction)

Specification document

[aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/changes-for-2022](https://www.aqa.org.uk/subjects/mathematics/aqa-certificate/further-mathematics-8365/changes-for-2022)

Resources for the current specification including specimen, practice and past papers

[allaboutmaths.aqa.org.uk/1644](https://allaboutmaths.aqa.org.uk/1644)

AQA student resources – worksheets for most topics

[allaboutmaths.aqa.org.uk/1649](https://allaboutmaths.aqa.org.uk/1649)

Exampro for L2 Further Maths – currently free for 2022

[exampro.co.uk/mathematics/#1628868660003-cfee5949-a590](https://www.exampro.co.uk/mathematics/#1628868660003-cfee5949-a590)

Legacy resources – very similar specification so still useful including past papers

[allaboutmaths.aqa.org.uk/level2FM](https://allaboutmaths.aqa.org.uk/level2FM)

Route maps and teaching guidance

[allaboutmaths.aqa.org.uk/1647](https://allaboutmaths.aqa.org.uk/1647)

[allaboutmaths.aqa.org.uk/1648](https://allaboutmaths.aqa.org.uk/1648)

## External resources for Level 2 Further Mathematics

Corbett Maths – lots of videos and worksheets

[corbettmaths.com/more/further-maths/](https://corbettmaths.com/more/further-maths/)

Increasingly difficult questions – nice resource for extension questions. Covers GCSE also

[taylorda01.weebly.com/increasingly-difficult-questions.html#l2fm](https://taylorda01.weebly.com/increasingly-difficult-questions.html#l2fm)

Mr Barton – selection of worksheets and PowerPoints

[mrbartonmaths.com/students/aqa-level-2-certificate-in-further-mathematics/](http://www.mrbartonmaths.com/students/aqa-level-2-certificate-in-further-mathematics/)

Advanced Maths Support Program (government funded) also produces resources. Teacher access is free for schools who register

[amsp.org.uk/teachers/11-16-maths/resources](https://amsp.org.uk/teachers/11-16-maths/resources)

Notes

Contact us

Our friendly team will be happy to support you between 8am and 5pm, Monday to Friday.

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