



# General Certificate of Secondary Education

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Mathematics

9307

*Pilot Specification*

2009

Material accompanying this Specification

- Specimen Assessment Materials 2009
- Teacher's Guide and Teaching Resource 2008
- Further exemplar material 2009

# SPECIFICATION

Vertical black lines indicate a significant change or addition to the specification published for 2008.

Further copies of this specification booklet are available from:

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# Contents

## Background Information

|          |  |   |
|----------|--|---|
| <b>1</b> | Introduction                                       | 4 |
| <b>2</b> | Specification at a Glance                          | 6 |
| <b>3</b> | Availability of Assessment Units and Entry Details | 7 |

## Scheme of Assessment

|          |                       |    |
|----------|-----------------------|----|
| <b>4</b> | Introduction          | 9  |
| <b>5</b> | Aims                  | 10 |
| <b>6</b> | Assessment Objectives | 10 |
| <b>7</b> | Scheme of Assessment  | 11 |

## Subject Content

|           |                            |    |
|-----------|----------------------------|----|
| <b>8</b>  | Summary of Subject Content | 14 |
| <b>9</b>  | Subject Content Unit 2     | 16 |
| <b>10</b> | Subject Content Unit 3     | 32 |

## Other Issues

|           |  |    |
|-----------|--|----|
| <b>11</b> | Spiritual, Moral, Ethical, Social, Cultural and Other Issues | 55 |
|-----------|--|----|

## Awarding and Reporting

|           |                                 |    |
|-----------|---------------------------------|----|
| <b>12</b> | Grading, Shelf-life and Re-sits | 57 |
|-----------|---------------------------------|----|

## Appendices

|          |                                    |    |
|----------|------------------------------------|----|
| <b>A</b> | Functional Mathematics             | 60 |
| <b>B</b> | Formulae Sheets                    | 63 |
| <b>C</b> | Overlaps with Other Qualifications | 65 |

# Background Information

## 1

### Introduction

As part of the 14–19 Curriculum Pathways Project, AQA has been contracted by QCA to develop and pilot a GCSE in Mathematics. A pilot qualification based on this revised specification will be available for award in summer 2009. First teaching to this specification commenced in September 2007. Developments related to the new Key Stage 4 Programme of Study and the pilot assessment principles for functional skills have led to minor changes in this specification for 2009.

#### 1.1 National Qualifications Framework

GCSE qualifications have the following equivalence in the National Qualifications Framework.

Grades A\* – C = Level 2

Grades D – G = Level 1

#### 1.2 Requirements at GCSE

##### ICT

The subject content of all GCSEs must require candidates to make effective use of ICT and provide, where appropriate, assessment opportunities for ICT. Details of how the teaching of this specification can encourage the application and development of ICT skills are given in Section 11. However, ICT skills are not assessed by any component of this specification.

##### Functional Skills

This specification is required to assess the functional process skills in mathematics as these are a part of the *Programme of Study: Mathematics (Key Stage 4)*. More details can be found in Section 6.2 of this specification

##### Communication

All GCSE specifications must ensure that the assessment arrangements require that, when they produce extended written material, candidates have to:

- ensure that text is legible and that spelling, punctuation and grammar are accurate so that meaning is clear
- present information in a form that suits its purpose
- use a suitable structure and style of writing.

|                            |   |
|----------------------------|---|
| Tiering                    | <p>The stand alone Functional Mathematics assessment referred to in this specification is at Level 2. A stand-alone assessment of Functional Mathematics at Level 1 is also available.</p> <p>The scheme of assessment must include question papers targeted at two tiers of grades: A* – D (Higher), and C – G (Foundation).</p> <p>Candidates should be entered at the tier appropriate to their attainment. Candidates who fail to achieve the mark for the lowest grade available at Foundation tier will be recorded as unclassified (U). There is a safety net for candidates entered for the Higher tier, where an allowed grade E will be awarded for the unit where candidates just fail to achieve grade D.</p> |
| Citizenship                | <p>From 2002, students in England have been required to study Citizenship as a national curriculum subject. Each GCSE specification must signpost, where appropriate, opportunities for developing citizenship knowledge, skills and understanding. Further details for this specification are given in Section 11.</p>   |
| Other Issues               | <p>All specifications must identify ways in which the study of the subject can contribute to developing understanding of spiritual, moral, ethical, social and cultural issues, European developments, environmental issues, and health and safety. Further details for this specification are given in Section 11.</p>   |
| Wales and Northern Ireland | <p>Centres in Northern Ireland/Wales must refer to the Statement in Section 8.1 of this specification.</p>  |

## Specification at a Glance *Mathematics*

- This is one of two pilot GCSE Mathematics specifications offered by AQA. This specification is a modular specification leading to a GCSE in Mathematics. There is also a pilot specification for GCSE in Additional Mathematics which is a linear specification.
- A Level 2 assessment of Functional Mathematics is an expectation of this pilot specification. This qualification is entered separately and the achievement in this unit is awarded separately as Level 2, Level 1 or Unclassified. It does not contribute to the overall grading of GCSE mathematics. Further information about the assessment of Functional mathematics can be found in Appendix of this specification and in AQA pilot specification 9305: Functional Skills Certificate in Mathematics.
- There are two tiers of assessment for GCSE, Foundation (C – G) and Higher (A\* – D).

### GCSE Mathematics Pilot (9307)

Although Functional mathematics is no longer a part of this specification, AQA has retained the entry codes and titles of the GCSE Units as Units 2 and 3

#### Unit 2

##### Number and Statistics

Written Paper 1/3 of the total  
GCSE Mathematics assessment

2 × 30 minutes (Both tiers)

Section A – Calculator

Section B – Non-calculator

#### Unit 3

##### Geometry and measures and Algebra

2 Written Papers 2/3 of the total  
GCSE Mathematics assessment

60 minutes each (Both tiers)

Paper 1 – Non-calculator

Paper 2 – Calculator

## 3

## Availability of Assessment Units and Entry Details

### 3.1 Availability of Assessment Units

Examinations will be available as follows:

| Availability of Units |   |   | Availability of Qualification |
|-----------------------|---|---|-------------------------------|
| Unit                  | 2 | 3 |                               |
| January 2009          | ✓ | ✓ | ✓                             |
| June 2009             | ✓ | ✓ | ✓                             |
| January 2010          | ✓ | ✓ | ✓                             |
| June 2010             | ✓ | ✓ | ✓                             |

Examinations and qualifications will continue to be available on this pilot until a revised national qualification in GCSE Mathematics becomes available. At present this is expected to be 2012.

### 3.2 Entry Codes

Normal entry requirements apply, but the following information should be noted.

This is a pilot specification and, as such, centres will be invited to take part with the maximum number of candidates and centres determined by the regulator.

The **Subject Code** for entry to the GCSE award is 9307.

A separate entry is needed for each of the two units. Both papers of Unit 3 must be taken in the same series.

More detailed information, including component codes will be issued to pilot centres separately.

Information about re-taking units is given in Section 13 of this specification.

### 3.3 Classification Codes

Each specification is assigned to a national classification code, indicating the subject area to which it belongs.

Centres should be aware that candidates, who enter for more than one GCSE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification code for this specification is 2210.

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|     |                          |   |
|-----|--------------------------|---|
| 3.4 | Private Candidates       | This specification is not available for private candidates.   |
| 3.5 | Special Consideration    | <p>Special consideration may be requested for candidates whose work has been affected by illness or other exceptional circumstances. The appropriate form and all relevant information should be forwarded to the AQA office which deals with such matters for the centre concerned.</p> <p>Details are available from the AQA Website (<a href="http://www.aqa.org.uk">www.aqa.org.uk</a>). Centres contacting AQA should ask for a copy of <i>Regulations and Guidance relating to Candidates who are Eligible for Adjustments in Examinations</i>.</p> |
| 3.6 | Access Arrangements      | <p>Special arrangements may be provided for candidates with special needs.</p> <p>Details are available from the AQA Website (<a href="http://www.aqa.org.uk">www.aqa.org.uk</a>). Centres contacting AQA should ask for the Candidate Services Department and a copy of <i>Regulations and Guidance relating to Candidates who are Eligible for Adjustments in Examinations</i>.</p>   |
| 3.7 | Language of Examinations | All components are provided in English only.  |

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# Scheme of Assessment

## 4

## Introduction

### 4.1 National Criteria

This GCSE Mathematics Specification complies with the following:

The Operating Rules for Pilot GCSE Mathematics

The GCSE and GCE A/AS Code of Practice;

The GCSE Qualification Specific Criteria;

The Arrangements for the Statutory Regulation of External Qualifications in England, Wales and Northern Ireland: Common Criteria.

### 4.2 Rationale

AQA offers a suite of qualifications for GCSE Mathematics. Specification A is a traditional linear scheme. Specification B is a modular scheme suitable for both pre-16 and post-16 candidates. This pilot scheme is a unit-based qualification which shares many of the features of flexibility from the existing modular scheme.

Mathematics is essentially a holistic subject and, as such, should be taught in this way with connections being made between the sections on *Number and algebra*, *Geometry and measures* and *Statistics* as required in the *Programme of Study: Mathematics (Key Stage 4)*. This pilot specification and the specification for Functional Mathematics (9305) allow candidates to take the untiered Functional Mathematics assessment and the two tiered units at the most appropriate time for them with four opportunities over a two-year course.

### 4.3 Prior Level of Attainment and Recommended Prior Learning

There is progression of material through all levels at which the subject is studied. This specification therefore builds on the Key Stage 3 Programme of Study.

It is also expected that candidates will have reached the required level of literacy through study at Key Stage 3.

### 4.4 Progression

This qualification is a recognised part of the National Qualifications Framework and is being developed to fit into the whole 14–19 Pathways programme. As such, GCSE Mathematics provides progression from Key Stage 3 to GCE A/AS Mathematics or further study at Advanced or Advanced Subsidiary level in other subjects or further study at Level 3, or directly into employment.

## 5

### Aims

The aims set out below are consistent with the revised *Programme of study: Mathematics (Key Stage 4)*.

These aims describe the educational rationale for students following a course in mathematics at GCSE level. They should be read in conjunction with the associated assessment objectives since not all of the aims can be translated into measurable outcomes or objectives.

A course based on this specification should enable candidates to

- a develop a positive attitude to mathematics
- b develop knowledge, skills and understanding in mathematical methods and concepts
- c acquire the basic strategies for problem solving
- d acquire foundations for the further study of mathematics and other disciplines
- e have the confidence to select and apply the mathematical techniques and methods they have learned in everyday and real-world situations, including unfamiliar contexts
- f be able to make deductions and inferences and draw conclusions
- g interpret and communicate mathematical information in variety of forms appropriate to the information and context, including the use of ICT
- h develop an awareness of the importance of mathematics to society.

## 6

### Assessment Objectives

#### 6.1 Assessment Objectives

This specification requires candidates to

AO1 *Demonstrate knowledge, skills and understanding*

AO2 *Apply knowledge and understanding using appropriate terms, concepts and methods in abstract and real-life contexts*

AO3 *Demonstrate strategies for problem-solving.*

## 6.2 Functional Skills

This specification provides opportunities for developing the process skills for Level 2 as defined in the *Functional Skills Standards: Mathematics*. These skills are tested within the stand-alone Functional Mathematics assessment but also throughout the GCSE units. As this pilot specification evolves, it is anticipated that 50% of the marks available in the Foundation tier GCSE and 20% of the Higher tier will assess the functional skills. The style of question will, however, differ across the two qualifications.

## 7

## Scheme of Assessment

## 7.1 Assessment Units

The Scheme of Assessment comprises two units as detailed below:

Unit 2  
Number and Statistics

|  |                              |
|--|------------------------------|
| Written Paper                          | (Section A – Calculator)     |
|  | (Section B – Non-calculator) |
| Foundation Tier                        | 2 × 30 minutes               |
| Higher Tier                            | 2 × 30 minutes               |
| 1/3 of the GCSE Mathematics assessment | 2 × 25 marks                 |

This written paper assesses Number and Statistics content. Assesses all Assessment Objectives of this specification. All questions are compulsory. A question/paper answer booklet will be provided.

Unit 3  
Geometry and Measures and  
Algebra

|  |        |          |
|--|--------|----------|
| Written Paper                          |        |          |
| Paper 1 (Non-calculator)               |        |          |
| Foundation Tier                        | 1 hour | 50 marks |
| Higher Tier                            | 1 hour | 50 marks |
| 1/3 of the GCSE Mathematics assessment |        |          |

|  |        |          |
|--|--------|----------|
| Written Paper                          |        |          |
| Paper 2 (Calculator)                   |        |          |
| Foundation Tier                        | 1 hour | 50 marks |
| Higher Tier                            | 1 hour | 50 marks |
| 1/3 of the GCSE Mathematics assessment |        |          |

Both papers assess the Geometry and measures and Algebra content. The Foundation tier will also include some number content. Assesses all Assessment Objectives of this specification. All questions are compulsory. A question/paper answer booklet will be provided.

Full details of the division of content across Units 2 and 3 are given in section 9 of this specification.

**7.2 Weighting of Assessment Objectives**

The approximate relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the overall Scheme of Assessment is shown in the following table:

| Assessment Objectives | Weighting of AOs (%) |
|-----------------------|----------------------|
| AO1                   | 45 – 60              |
| AO2                   | 25 – 40              |
| AO3                   | 10 – 20              |
|                       | <b>100</b>           |

**7.3 Weighting of Content**

The approximate relationship between the relative percentage weighting of the subject content and the overall Scheme of Assessment is shown in the following table:

| Content Area              | Weighting (%) |
|---------------------------|---------------|
| Number and Algebra (N)    | 45 – 55       |
| Geometry and Measures (G) | 25 – 35       |
| Statistics (S)            | 15 – 25       |
|                           | <b>100</b>    |

**7.4 Weighting across units**

The relationship between the relative percentage weighting of the Assessment Objectives (AOs) and the subject content by unit and by qualification is shown in the following table. These figures are approximate and may vary in the actual assessments.

|  | Unit and Content Weightings (%) |           |           |           |           |           |
|--|---------------------------------|-----------|-----------|-----------|-----------|-----------|
|  | AO1                             | AO2       | AO3       | N         | G         | S         |
| Unit 2                                     | 18                              | 10        | 5         | 14        | –         | 20        |
| Unit 3                                     | 37                              | 20        | 10        | 40        | 26        | –         |
| <b>Overall weighting of GCSE Units (%)</b> | <b>55</b>                       | <b>30</b> | <b>15</b> | <b>54</b> | <b>26</b> | <b>20</b> |

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**7.5 Written Papers**

The written papers for Units 2 and 3 at both tiers are designed so that 50% of the marks are focussed on the lowest two grades available. 25 – 30% of marks will focus on the highest two grades.

Common questions will be set on papers across the two tiers.

Formulae sheets for Foundation and Higher tier papers are provided in Appendix B and on page 2 of each examination paper for Unit 3.

The use of a calculator is not permitted in the following components: Unit 2 Section B, Unit 3 Paper 1. Slide rules, logarithmic tables and all other calculating aids are also forbidden. In Unit 2 Section A and Unit 3 Paper 2 candidates will be required to demonstrate the effective use of a calculator.

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**7.6 Calculators**

Candidates will be expected to have a scientific calculator. The calculator should have the following as a minimum requirement:

four rules and a square, square root, reciprocal and power function, brackets, a memory facility and appropriate exponential, trigonometric and statistical functions.

Further guidance on regulations relating to calculators can be obtained from *Instructions on the Conduct of Examinations*.

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**7.7 Entry Policy**

Centres are encouraged to enter candidates aiming to achieve grades D, E, F and G for the Foundation tier and grades A\*, A, B and C for the Higher tier.

# Subject Content

## 8

## Summary of Subject Content

### 8.1 Introduction

There are two tiers of entry for GCSE Mathematics candidates: Foundation and Higher. In the National Curriculum, published in 1999, the Key Stage 4 Programme of Study was divided into two tiers. The revised *Programme of study: Mathematics (Key Stage 4)* is not tiered but, for the sake of clarity, this specification uses the division of content into two tiers which is common to all present GCSE Mathematics specifications. Thus:

- the subject content unique to the Foundation tier is based on the Foundation Programme of Study
- the subject content common to both tiers and of the Higher tier only is based on the Higher Programme of Study
- in general, the Higher tier content of the specification subsumes the Foundation tier content.

This GCSE Specification has been written against the Key Stage 4 Programme of Study for England. Candidates entering for this GCSE in Northern Ireland and Wales must be taught all the material required by the National Curriculum in their own country.

### 8.2 Assessment Objectives

Within this specification the subject content is presented under the following areas.

#### Number and algebra (N)

- 1 Using, applying and problem-solving with number and algebra
- 2 Real numbers and their different representations
- 3 Calculations and manipulations with real numbers
- 4 Solving numerical problems
- 5 Equations, formulae and identities
- 6 Sequences, functions and graphs

#### Geometry and measures (G)

- 1 Using, applying and problem-solving with geometry and measures
- 2 Properties of 2-D and 3-D shapes
- 3 Transformations and coordinates
- 4 Measures and construction

**Statistics (S)**

- 1 Using, applying and problem-solving with statistics
- 2 Specifying the problem and planning
- 3 Collecting data
- 4 Processing and representing data
- 5 Interpreting and discussing results

All Assessment Objectives are assessed using this content.

---

**8.3 Breadth of Study**

In addition to the required knowledge, skills and understanding, the revised *Programme of Study: Mathematics (Key Stage 4)* also specifies key concepts (competence, creativity, applications and implications and critical understanding) that underpin the study of mathematics and key processes (representing, analysing, interpreting and evaluating) that students need to learn to make progress. Not all of these can be translated into measurable outcomes or objectives.

# 9 Subject Content

## Unit 2

### Number and algebra (N)

#### 1 Using, applying and problem-solving with number and algebra (A02, A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Problem solving

|                |  |       |   |
|----------------|--|-------|---|
| FN.1a          | select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems  | HN.1a | select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation       |
| HN.1b          | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches                      | HN.1b | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches |
| FN.1b<br>HN.1c | break down a complex calculation into simpler steps before attempting to solve it  | HN.1c | break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods   |
| FN.1c          | use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem |       |   |
| FN.1d          | make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy                                    | HN.1d | make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations  |

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Communicating**

|       |   |       |  |
|-------|---|-------|--|
| FN.1e | interpret and discuss numerical and algebraic information presented in a variety of forms   | HN.1e | discuss their work and explain their reasoning using an increasing range of mathematical language and notation   |
| FN.1f | use notation and symbols correctly and consistently within a given problem  | HN.1h | use notation and symbols correctly and consistently within a given problem   |
| FN.1g | use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem | HN.1f | use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem |
| FN.1h | present and interpret solutions in the context of the original problem  | HN.1g | present and interpret solutions in the context of the original problem   |
| FN.1i | review and justify their choice of mathematical presentation  | HN.1i | examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument  |

**Reasoning**

|       |   |       |   |
|-------|---|-------|---|
| FN.1j | explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems | HN.1j | explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems |
| FN.1k | show step-by-step deduction in solving a problem  | HN.1l | show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning   |
| FN.1l | understand the difference between a practical demonstration and a proof   | HN.1k | understand the difference between a practical demonstration and a proof   |

| Foundation Tier |   | Higher Tier |   |
|-----------------|---|-------------|---|
| FN.1m           | recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem | HN.1m       | recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem |

## 2 Real numbers and their different representations (A01, A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Integers

|       |   |
|-------|---|
| HN.2a | use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition |
|-------|---|

### Powers and roots

|       |  |
|-------|--|
| HN.2b | use the terms square, positive square root, negative square root, cube and cube root; use index notation and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display |
|-------|--|

## Foundation Tier

## Higher Tier

**Fractions**

|       |   |       |   |
|-------|---|-------|---|
| FN.2c | understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator | HN.2c | understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator |
|-------|---|-------|---|

**Decimals**

|                |   |       |   |
|----------------|---|-------|---|
| FN.2d<br>HN.2d | use decimal notation and recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals | HN.2d | use decimal notation and recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals |
|----------------|---|-------|---|

**Percentages**

|       |  |                |  |
|-------|--|----------------|--|
| FN.2e | understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of'; use percentage in real-life situations | FN.2e<br>HN.2e | understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of'; use percentage in real-life situations |
|-------|--|----------------|--|

**Ratio**

|       |   |       |   |
|-------|---|-------|---|
| FN.2f | use ratio notation, including reduction to its simplest form and its various links to fraction notation | HN.2f | use ratio notation, including reduction to its simplest form and its various links to fraction notation |
|-------|---|-------|---|

### 3 Calculations and manipulations with real numbers (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Number operations and the relationships between them

|       |  |       |   |
|-------|--|-------|---|
|       |  | HN.3a | multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power $n$ is raising the result of this operation to power $\frac{1}{n}$ |
|       |  | HN.3b | use brackets and the hierarchy of operations  |
| FN.3c | calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal | HN.3c | calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction   |

| Foundation Tier |  | Higher Tier |  |
|-----------------|--|-------------|--|
| FN.3d<br>HN.3d  | understand and use unit fractions as multiplicative inverses; multiply and divide a fraction by an integer, by a unit fraction and by a general fraction | HN.3d       | understand and use unit fractions as multiplicative inverses; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction   |
| FN.3e           | convert simple fractions of a whole to percentages of the whole and vice versa, then understand the multiplicative nature of percentages as operators    | HN.3e       | convert simple fractions of a whole to percentages of the whole and vice versa; then understand the multiplicative nature of percentages as operators; calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems |
| FN.3f           | divide a quantity in a given ratio   | HN.3f       | divide a quantity in a given ratio   |

**Mental methods**

|                |  |       |   |
|----------------|--|-------|---|
| FN.3g<br>HN.3g | recall all positive integer complements to 100; recall all multiplication facts to $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall integer squares from $11 \times 11$ to $15 \times 15$ and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions | HN.3g | recall integer squares from $2 \times 2$ to $15 \times 15$ and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$ , the corresponding rule for negative numbers, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$ |
|                |  | HN.3h | round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations, converting to standard index form to make sensible estimates for calculations involving multiplication and/or division               |

| Foundation Tier |   | Higher Tier |  |
|-----------------|---|-------------|--|
| FN.3i           | develop a range of strategies for mental calculation; derive unknown facts from those they know; add and subtract mentally numbers with up to two decimal places; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments | FN.3i       | develop a range of strategies for mental calculation; add and subtract mentally numbers with up to two decimal places; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments |

**Written methods**

|       |  |       |   |
|-------|--|-------|---|
| FN.3j | use standard column procedures for addition and subtraction of integers and decimals   |       |   |
| FN.3k | use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to 2 d.p.) by transforming it to a problem involving division by an integer | FN.3k | division by decimal (up to 2 d.p.) by division using an integer; understand where to position the decimal point by considering what happens if they multiply equivalent fractions, eg, given that...work out... |
| FN.3l | use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer  | HN.3i | use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer       |
| FN.3m | solve simple percentage problems, including increase and decrease  | HN.3j | solve percentage problems, including percentage increase and decrease; and reverse percentages  |
| FN.3n | solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution   | FN.3n | solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution  |

| Foundation Tier | Higher Tier |  |
|-----------------|-------------|--|
|                 | HN.3k       | represent repeated proportional change using a multiplier raised to a power  |
|                 | HN.3l       | calculate an unknown quantity from quantities that vary in direct or inverse proportion  |
|                 | HN.3m       | calculate with standard index form   |
|                 | HN.3n       | use surds and $\pi$ in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ |

**Calculator methods**

|                |  |       |   |
|----------------|--|-------|---|
| FN.3o          | use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers  | HN.3o | use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometrical and statistical functions relevant across this programme of study |
| FN.3p          | enter a range of calculations, including those involving standard index form and measures  | FN.3p | enter a range of calculations, including those involving measures   |
| FN.3q<br>HN.3p | understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation | HN.3p | understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation                              |
|                |  | HN.3q | use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements   |
|                |  | HN.3r | use standard index form display and know how to enter numbers in standard index form  |

| Foundation Tier | Higher Tier |   |
|-----------------|-------------|---|
|                 | HN.3s       | use calculators for reverse percentage calculations by doing an appropriate division          |
|                 | HN.3t       | use calculators to explore exponential growth and decay, using a multiplier and the power key |

#### 4 Solving numerical problems (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier  |   |
|-----------------|--|---|
|                 | HN.4a  | draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation |
| FN.4c<br>HN.4b  | estimate answers to problems; use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude                                 | HN.4b   |
| FN.4d           | give solutions in the context of the problem to an appropriate degree of accuracy, interpreting the solution shown on a calculator display, and recognising limitations on the accuracy of data and measurements | check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements  |

## Statistics (S)

### 1 Using, applying and problem-solving with statistics (A02,A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Problem solving

|                |   |       |   |
|----------------|---|-------|---|
| FS.1a          | <p>carry out each of the four aspects of the handling data cycle to solve problems:</p> <ul style="list-style-type: none"> <li>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed</li> <li>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources</li> <li>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem</li> <li>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data</li> </ul> | HS.1a | <p>carry out each of the four aspects of the handling data cycle to solve problems:</p> <ul style="list-style-type: none"> <li>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed</li> <li>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources</li> <li>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem</li> <li>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data</li> </ul> |
| FS.1b<br>HS.1b | <p>identify what further information is needed to pursue a particular line of enquiry; select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)</p>  | HS.1b | <p>select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)</p>  |

| Foundation Tier |   | Higher Tier |  |
|-----------------|---|-------------|--|
| FS.1c           | select and organise the appropriate mathematics and resources to use for a task |             |  |
| FS.1d           | review progress while working; check and evaluate solutions                     |             |  |

**Communicating**

|       |  |       |   |
|-------|--|-------|---|
| FS.1e | interpret, discuss and synthesise information presented in a variety of forms                          |       |   |
| FS.1f | communicate mathematically, including using ICT, making use of diagrams and related explanatory text   | HS.1c | communicate mathematically, with emphasis on the use of an increasing range of diagrams and related explanatory text, on the selection of their mathematical presentation, explaining its purpose and approach, and on the use of symbols to convey statistical meaning |
| FS.1g | examine critically, and justify, their choices of mathematical presentation of problems involving data |       |   |

**Reasoning**

|                |   |       |   |
|----------------|---|-------|---|
| FS.1h          | apply mathematical reasoning, explaining and justifying inferences and deductions                   | HS.1d | apply mathematical reasoning, explaining and justifying inferences and deductions, justifying arguments and solutions |
| HS.1e          | identify exceptional or unexpected cases when solving statistical problems                          | HS.1e | identify exceptional or unexpected cases when solving statistical problems  |
| FS.1i<br>HS.1f | explore connections in mathematics and look for relationships between variables when analysing data | HS.1f | explore connections in mathematics and look for relationships between variables when analysing data                   |

| Foundation Tier |  | Higher Tier |  |
|-----------------|--|-------------|--|
| FS.1j           | recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis | HS.1g       | recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis |

## 2 Specifying the problem and planning (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier |  | Higher Tier |   |
|-----------------|--|-------------|---|
| FS.2a           | see that random processes are unpredictable  | HS.2a       | see that random processes are unpredictable   |
| FS.2b<br>HS.2b  | identify key questions that can be addressed by statistical methods  | HS.2b       | identify key questions that can be addressed by statistical methods   |
| FS.2c           | discuss how data relate to a problem, identify possible sources of bias and plan to minimise it  | HS.2c       | discuss how data relate to a problem, identify possible sources of bias and plan to minimise it   |
| FS.2d           | identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals | HS.2d       | identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals; select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling |
| FS.2e<br>HS.2e  | design an experiment or survey; decide what primary and secondary data to use  | HS.2e       | design an experiment or survey; decide what primary and secondary data to use   |

### 3 Collecting data (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier |   | Higher Tier |  |
|-----------------|---|-------------|--|
| FS.3a           | design and use data-collection sheets for grouped, discrete and continuous data; collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys | HS.3a       | collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys |
| FS.3b           | gather data from secondary sources, including printed tables and lists from ICT-based sources   | HS.3b       | gather data from secondary sources, including printed tables and lists from ICT-based sources                              |
| FS.3c           | design and use two-way tables for discrete and grouped data   | HS.3c       | design and use two-way tables for discrete and grouped data  |
|                 |   | HS.3d       | deal with practical problems such as non-response or missing data  |

### 4 Processing and representing data (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier |  | Higher Tier |   |
|-----------------|--|-------------|---|
| FS.4a           | draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs for time series, scatter graphs, frequency diagrams and stem-and-leaf diagrams | HS.4a       | draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs (time series), scatter graphs, frequency diagrams, stem-and-leaf diagrams, cumulative frequency tables and diagrams, box plots and histograms for grouped continuous data |
| FS.4b           | calculate mean, range and median of small data sets with discrete then continuous data; identify the modal class for grouped data  |             |   |

| Foundation Tier |   | Higher Tier |  |
|-----------------|---|-------------|--|
| FS.4c           | understand and use the probability scale  |             |  |
| FS.4d           | understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency | HS.4b       | understand and use estimates or measures of probability from theoretical models, or from relative frequency  |
| FS.4e           | list all outcomes for single events, and for two successive events, in a systematic way   | HS.4c       | list all outcomes for single events, and for two successive events, in a systematic way  |
| FS.4f           | identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1                            | HS.4d       | identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1   |
| FS.4g           | find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data                                 | HS.4e       | find the median, quartiles and interquartile range for large data sets and calculate the mean for large data sets with grouped data  |
|                 |   | HS.4f       | calculate an appropriate moving average  |
|                 |   | HS.4g       | know when to add or multiply two probabilities: if $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$ , whereas if $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ |
|                 |   | HS.4h       | use tree diagrams to represent outcomes of compound events, recognising when events are independent  |
| FS.4h           | draw lines of best fit by eye, understanding what these represent   | HS.4i       | draw lines of best fit by eye, understanding what these represent  |
| HS.4j           | use relevant statistical functions on a calculator or spreadsheet   | HS.4j       | use relevant statistical functions on a calculator or spreadsheet  |

## 5 Interpreting and discussing results (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier |   | Higher Tier |   |
|-----------------|---|-------------|---|
| FS.5a           | relate summarised data to the initial questions   | HS.5a       | relate summarised data to the initial questions   |
| FS.5b           | interpret a wide range of graphs and diagrams and draw conclusions  | HS.5b       | interpret a wide range of graphs and diagrams and draw conclusions; identify seasonality and trends in time series  |
| FS.5c           | look at data to find patterns and exceptions  | HS.5c       | look at data to find patterns and exceptions  |
| FS.5d           | compare distributions and make inferences, using the shapes of distributions and measures of average and range  | HS.5d       | compare distributions and make inferences, using shapes of distributions and measures of average and spread, including median and quartiles; understand frequency density   |
| FS.5e           | consider and check results and modify their approach if necessary   | HS.5e       | consider and check results, and modify their approach if necessary  |
| FS.5f<br>HS.5f  | appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply ‘no relationship’ but merely ‘no linear relationship’ | HS.5f       | appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply ‘no relationship’ but merely ‘no linear relationship’ |
| FS.5g           | use the vocabulary of probability to interpret results involving uncertainty and prediction   | HS.5g       | use the vocabulary of probability to interpret results involving uncertainty and prediction   |
| FS.5h           | compare experimental data and theoretical probabilities   | HS.5h       | compare experimental data and theoretical probabilities   |
| FS.5i           | understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics   | HS.5i       | understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population parameters  |

| Foundation Tier |   | Higher Tier |   |
|-----------------|---|-------------|---|
| FS.5j           | discuss implications of findings in the context of the problem                    |             |   |
| FS.5k           | interpret social statistics including index numbers; time series; and survey data | FS.5k       | interpret social statistics including index numbers; time series; and survey data |

# 10 Subject Content

## Unit 3

### 1 Using, applying and problem-solving with number and algebra (A02, A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Problem solving

|                |  |       |   |
|----------------|--|-------|---|
| FN.1a          | select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems  | HN.1a | select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation       |
| HN.1b          | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches                      | HN.1b | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches |
| FN.1b<br>HN.1c | break down a complex calculation into simpler steps before attempting to solve it  | HN.1c | break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods   |
| FN.1c          | use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem |       |   |
| FN.1d          | make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy                                    | HN.1d | make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations  |

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Communicating**

|       |   |       |  |
|-------|---|-------|--|
| FN.1e | interpret and discuss numerical and algebraic information presented in a variety of forms   | HN.1e | discuss their work and explain their reasoning using an increasing range of mathematical language and notation   |
| FN.1f | use notation and symbols correctly and consistently within a given problem  | HN.1h | use notation and symbols correctly and consistently within a given problem   |
| FN.1g | use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem | HN.1f | use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem |
| FN.1h | present and interpret solutions in the context of the original problem  | HN.1g | present and interpret solutions in the context of the original problem   |
| FN.1i | review and justify their choice of mathematical presentation  | HN.1i | examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument  |

**Reasoning**

|       |   |       |   |
|-------|---|-------|---|
| FN.1j | explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems | HN.1j | explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems |
| FN.1k | show step-by-step deduction in solving a problem  | HN.1l | show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning   |
| FN.1l | understand the difference between a practical demonstration and a proof   | HN.1k | understand the difference between a practical demonstration and a proof   |

| Foundation Tier |   | Higher Tier |   |
|-----------------|---|-------------|---|
| FN.1m           | recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem | HN.1m       | recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem |

## 2 Real numbers and their different representations (A01, A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Integers

|                |  |
|----------------|--|
| FN.2a<br>HN.2a | use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers and negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition |
|----------------|--|

### Powers and roots

|                |  |
|----------------|--|
| FN.2b<br>HN.2b | use the terms square, positive and negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; use index laws for multiplication and division of integer powers; express standard index form both in conventional notation and on a calculator display |
|----------------|--|

### 3 Calculations and manipulations with real numbers (A01, A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Number operations and the relationships between them

|                |  |       |  |
|----------------|--|-------|--|
| FN.3a<br>HN.3a | add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers; use inverse operations |       |  |
| FN.3b          | use brackets and the hierarchy of operations   | HN.3b | use brackets and the hierarchy of operations |

#### Mental methods

|                |  |       |   |
|----------------|--|-------|---|
| FN.3g<br>HN.3g | recall all positive integer complements to 100; recall all multiplication facts to $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall integer squares from $11 \times 11$ to $15 \times 15$ and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions | HN.3g | recall integer squares from $2 \times 2$ to $15 \times 15$ and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$ , the corresponding rule for negative numbers, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$ |
|----------------|--|-------|---|

| Foundation Tier |   | Higher Tier |   |
|-----------------|---|-------------|---|
| FN.3h           | round to the nearest integer and to one significant figure; estimate answers to problems involving decimals   | HN.3h       | round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know |
| FN.3i           | develop a range of strategies for mental calculation; derive unknown facts from those they know; add and subtract mentally numbers with up to two decimal places; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments |             |   |

**Written methods**

|       |  |       |  |
|-------|--|-------|--|
| FN.3j | use standard column procedures for addition and subtraction of integers and decimals   |       |  |
| FN.3k | use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to 2 d.p.) by transforming it to a problem involving division by an integer |       |  |
| HN.3n | use $\pi$ in exact calculations, without a calculator  | HN.3n | use surds and $\pi$ in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ |

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Calculator methods**

|                |  |       |   |
|----------------|--|-------|---|
| FN.3o          | use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers  | HN.3o | use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometrical and statistical functions relevant across this programme of study |
| FN.3p          | enter a range of calculations, including those involving standard index form and measures  | FN.3p | enter a range of calculations, including those involving measures   |
| FN.3q<br>HN.3p | understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation | HN.3p | understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation                              |

**4 Solving numerical problems (A01, A02 & A03)**

Pupils should be taught to:

| Foundation Tier | Higher Tier   |       |  |
|-----------------|---|-------|--|
| FN.4a<br>HN.4a  | draw on their knowledge of operations, inverse operations and the relationships between them, and of simple integer powers and their corresponding roots, and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, a range of measures and compound measures, metric units and conversion between metric and common imperial units, set in a variety of contexts | HN.4a | draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems including those involving surds, measures and conversion between measures, and compound measures defined within a particular situation |

| Foundation Tier |  | Higher Tier |  |
|-----------------|--|-------------|--|
| FN.4b           | select appropriate operations, methods and strategies to solve number problems, including trial and improvement where a more efficient method to find the solution is not obvious                                |             |  |
| FN.4c<br>HN.4b  | estimate answers to problems; use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude                                 | HN.4b       | check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements |
| FN.4d           | give solutions in the context of the problem to an appropriate degree of accuracy, interpreting the solution shown on a calculator display, and recognising limitations on the accuracy of data and measurements |             |  |

## 5 Equations, formulae and identities (A01, A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Use of symbols

|                |  |       |  |
|----------------|--|-------|--|
| FN.5a<br>HN.5a | distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities | HN.5a | distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities |
|----------------|--|-------|--|

| Foundation Tier |  | Higher Tier |  |
|-----------------|--|-------------|--|
| FN.5b<br>HN.5b  | understand that the transformation of algebraic expressions obeys and generalises the rules of generalised arithmetic, expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors; distinguish in meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’ | HN.5b       | understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic; expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors, factorising quadratic expressions including the difference of two squares and cancelling common factors in rational expressions |
|                 |  | HN.5c       | know the meaning of and use the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’   |

**Index notation**

|       |   |       |   |
|-------|---|-------|---|
| FN.5c | use index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ | HN.5d | use index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ |
|-------|---|-------|---|

**Inequalities**

|       |   |       |  |
|-------|---|-------|--|
| FN.5d | solve simple linear inequalities in one variable, and represent the solution set on a number line | HN.5j | solve linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set |
|-------|---|-------|--|

**Equations**

|       |   |       |   |
|-------|---|-------|---|
| HN.5e | set up simple equations; solve simple equations by using inverse operations or by transforming both sides in the same way | HN.5e | set up simple equations; solve simple equations by using inverse operations or by transforming both sides in the same way |
|-------|---|-------|---|

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Linear Equations**

|       |  |       |   |
|-------|--|-------|---|
| FN.5e | solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution | HN.5f | solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution |
|-------|--|-------|---|

**Formulae**

|       |  |       |  |
|-------|--|-------|--|
| FN.5f | use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols; substitute numbers into a formula; derive a formula and change its subject | HN.5g | use formulae from mathematics and other subjects; substitute numbers into a formula; change the subject of a formula including cases where the subject occurs twice, or where a power of the subject appears; generate a formula |
|-------|--|-------|--|

**Direct and inverse proportion**

|       |   |
|-------|---|
| HN.5h | set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations |
|-------|---|

**Simultaneous linear equations**

|       |  |
|-------|--|
| HN.5i | Find the exact solutions of two simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection |
|-------|--|

Foundation Tier

Higher Tier

**Quadratic equations**

|       |   |
|-------|---|
| HN.5k | solve quadratic equations by factorisation, completing the square and using the quadratic formula |
|-------|---|

**Simultaneous linear and quadratic equations**

|       |  |
|-------|--|
| HN.5l | solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, or where the second is of the form $x^2 + y^2 = r^2$ |
|-------|--|

**Numerical Methods**

|       |  |       |  |
|-------|--|-------|--|
| HN.5m | use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them | HN.5m | use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them |
|-------|--|-------|--|

## 6 Sequences, functions and graphs (AO1,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Sequences

|                |   |       |   |
|----------------|---|-------|---|
| FN.6a<br>HN.6a | generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); use linear expressions to describe the $n$ th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated | HN.6a | generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the $n$ th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated |
|----------------|---|-------|---|

### Graphs of linear functions

|       |   |       |   |
|-------|---|-------|---|
| FN.6b | use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ , or implicitly   | HN.6b | use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ , or implicitly; no table or axes given |
| FN.6c | construct linear functions from real-life problems and plot their corresponding graphs; discuss and interpret graphs modelling real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines; draw line of best fit through a set of linearly related points and find its equation | HN.6c | find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$ ); understand that the form $y = mx + c$ represents a straight line and that $m$ is the gradient of the line and $c$ is the value of the $y$ – intercept; explore the gradients of parallel lines and lines perpendicular to each other     |

## Foundation Tier

## Higher Tier

**Gradients**

|       |  |
|-------|--|
| FN.6e | find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$ ); investigate the gradients of parallel lines |
|-------|--|

**Interpret graphical information**

|       |  |
|-------|--|
| FN.6e | interpret information presented in a range of linear and non-linear graphs |
|-------|--|

|       |  |
|-------|--|
| HN.6d | construct linear functions and plot the corresponding graphs arising from real-life problems; discuss and interpret graphs modelling real situations |
|-------|--|

**Quadratic functions**

|       |   |
|-------|---|
| HN.6e | generate points and plot graphs of simple quadratic functions, then more general quadratic functions; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function |
|-------|---|

|       |  |
|-------|--|
| HN.6e | generate points and plot graphs of simple quadratic functions, then more general quadratic functions; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions |
|-------|--|

**Other functions**

|       |   |
|-------|---|
| HN.6f | plot graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$ , the exponential function $y = k^x$ for integer values of $x$ and simple positive values of $k$ , the circular functions $y = \sin x$ and $y = \cos x$ , using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions |
|-------|---|

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Transformation of functions**

|       |  |
|-------|--|
| HN.6g | apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$ , $y = f(ax)$ , $y = f(x + a)$ , $y = af(x)$ for linear, quadratic, sine and cosine functions $f(x)$ |
|-------|--|

**Loci**

|       |  |
|-------|--|
| HN.6h | construct the graphs of simple loci including the circle $x^2 + y^2 = r^2$ for a circle of radius $r$ centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle |
|-------|--|

# Geometry and measures (G)

## 1 Using, applying and problem-solving with geometry and measures (A02,A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Problem solving

|                |   |       |  |
|----------------|---|-------|--|
| FG.1a<br>HG.1a | select problem-solving strategies and resources, including ICT tools, to use in geometrical work, and monitor their effectiveness; consider and explain the extent to which the selections they made were appropriate | HG.1a | select the problem-solving strategies to use in geometrical work, and consider and explain the extent to which the selections they made were appropriate |
| FG.1b          | select and combine known facts and problem-solving strategies to solve complex problems   | HG.1b | select and combine known facts and problem-solving strategies to solve more complex geometrical problems   |
| FG.1c<br>HG.1c | identify what further information is needed to solve a geometrical problem; break complex problems down into a series of tasks; develop and follow alternative lines of enquiry                                       | HG.1c | develop and follow alternative lines of enquiry, justifying their decisions to follow or reject particular approaches                                    |

### Communicating

|                |  |       |   |
|----------------|--|-------|---|
| FG.1d          | interpret, discuss and synthesise geometrical information presented in a variety of forms  |       |   |
| FG.1e<br>HG.1d | communicate mathematically with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams | HG.1d | communicate mathematically, with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams |

| Foundation Tier |  | Higher Tier |  |
|-----------------|--|-------------|--|
| FG.1f           | use geometrical language appropriately                       | HG.1e       | use precise formal language and exact methods for analysing geometrical configurations |
| FG.1g           | review and justify their choices of mathematics presentation | FG.1g       | review and justify their choices of mathematics presentation                           |

**Reasoning**

|       |  |       |   |
|-------|--|-------|---|
| FG.1h | distinguish between practical demonstrations and proofs  | FG.1h | distinguish between practical demonstrations and proofs   |
| FG.1i | apply mathematical reasoning, explaining and justifying inferences and deductions  | HG.1f | apply mathematical reasoning, progressing from brief mathematical explanations towards full justifications in more complex contexts           |
|       |  | HG.1g | explore connections in geometry; pose conditional constraints of the type ‘If... then...’; and ask questions ‘What if...?’ or ‘Why?’          |
| FG.1j | show step-by-step deduction in solving a geometrical problem   | HG.1h | show step-by-step deduction in solving a geometrical problem  |
| FG.1k | state constraints and give starting points when making deductions  | HG.1i | state constraints and give starting points when making deductions   |
| FG.1l | recognise the limitations of any assumptions that are made; understand the effects that varying the assumptions may have on the solution | HG.1j | understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid |
| FG.1m | identify exceptional cases when solving geometrical problems   |       |   |

## 2 Properties of 2D and 3D shapes (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Angles

|       |  |
|-------|--|
| FG.2a | recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex |
| FG.2b | distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees   |

### Properties of triangles and other rectilinear shapes

|                |   |       |   |
|----------------|---|-------|---|
| FG.2c<br>HG.2a | distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices | HG.2a | distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices |
| FG.2d          | use angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of a quadrilateral is 360 degrees   | HG.2b | use angle properties of equilateral, isosceles and right-angled triangles; explain why the angle sum of a quadrilateral is 360 degrees  |
| FG.2e          | use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle  | FG.2e | use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle  |

| Foundation Tier |   | Higher Tier |  |
|-----------------|---|-------------|--|
| FG.2f<br>HG.2c  | recall the essential properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties | HG.2c       | recall the definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties |

**Properties of triangles and other rectilinear shapes**

|       |  |       |   |
|-------|--|-------|---|
| FG.2g | calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons | HG.2d | calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons  |
|       |  | HG.2e | understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions   |
| FG.2h | understand, recall and use Pythagoras’ theorem   | HG.2f | understand, recall and use Pythagoras’ theorem in 2-D, then 3-D problems; investigate the geometry of cuboids including cubes, and shapes made from cuboids, including the use of Pythagoras’ theorem to calculate lengths in three dimensions  |
|       |  | HG.2g | understand similarity of triangles and of other plane figures, and use this to make geometric inferences; understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings, then use these relationships in 3-D contexts, including finding the angles between a line and a plane (but not the angle between two planes or between two skew lines); calculate the area of a triangle using $\frac{1}{2}ab \sin C$ ; draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scalings in either or both the $x$ and $y$ directions; use the sine and cosine rules to solve 2-D and 3-D problems |

## Foundation Tier

## Higher Tier

**Properties of circles**

|       |   |       |   |
|-------|---|-------|---|
| FG.2i | recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that inscribed regular polygons can be constructed by equal division of a circle | HG.2h | recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; understand and use the fact that tangents from an external point are equal in length; explain why the perpendicular from the centre to a chord bisects the chord; understand that inscribed regular polygons can be constructed by equal division of a circle; prove and use the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180 degrees; prove and use the alternate segment theorem |
|-------|---|-------|---|

**3-D shapes**

|                |   |       |   |
|----------------|---|-------|---|
| FG.2j          | explore the geometry of cuboids (including cubes), and shapes made from cuboids   |       |   |
| FG.2k<br>HG.2i | use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms | HG.2i | use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms, pyramids, cylinders, cones and spheres; solve problems involving more complex shapes and solids, including segments of circles and frustums of cones |

### 3 Transformations and coordinates (AO1,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

#### Specifying transformations

|                        |  |              |  |
|------------------------|--|--------------|--|
| <p>FG.3a<br/>HG.3a</p> | <p>understand that rotations are specified by a centre and an (anticlockwise) angle; rotate a shape about the origin, or any other point; measure the angle of rotation using right angles, simple fractions of a turn or degrees; understand that reflections are specified by a mirror line, at first using a line parallel to an axis, then a mirror line such as <math>y = x</math> or <math>y = -x</math>; understand that translations are specified by a distance and direction (or a vector), and enlargements by a centre and positive scale factor</p> | <p>HG.3a</p> | <p>understand that rotations are specified by a centre and an (anticlockwise) angle; use any point as the centre of rotation; measure the angle of rotation, using right angles, fractions of a turn or degrees; understand that reflections are specified by a (mirror) line; understand that translations are specified by giving a distance and direction (or a vector), and enlargements by a centre and a positive scale factor</p> |
|------------------------|--|--------------|--|

#### Properties of transformations

|                        |   |              |   |
|------------------------|---|--------------|---|
| <p>FG.3b<br/>HG.3b</p> | <p>recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations</p> | <p>HG.3b</p> | <p>recognise and visualise rotations, reflections and translations including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations; use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations</p> |
| <p>FG.3c</p>           | <p>recognise, visualise and construct enlargements of objects using positive scale factors greater than one, then positive scale factors less than one; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not</p>   | <p>HG.3c</p> | <p>recognise, visualise and construct enlargements of objects; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not, then use positive fractional and negative scale factors</p>   |

| Foundation Tier |  | Higher Tier |   |
|-----------------|--|-------------|---|
| FG.3d           | recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments and apply this to triangles; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the implications of enlargement for area and for volume; distinguish between formulae for perimeter, area and volume by considering dimensions; understand and use simple examples of the relationship between enlargement and areas and volumes of shapes and solids | HG.3d       | recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the difference between formulae for perimeter, area and volume by considering dimensions; understand and use the effect of enlargement on areas and volumes of shapes and solids |

### Coordinates

|       |  |       |   |
|-------|--|-------|---|
| FG.3e | understand that one coordinate identifies a point on a number line, two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment AB, given points A and B, then calculate the length AB | HG.3e | understand that one coordinate identifies a point on a number line, that two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment AB, given the points A and B, then calculate the length AB |
|-------|--|-------|---|

### Vectors

|       |   |       |   |
|-------|---|-------|---|
| FG.3f | understand and use vector notation for translations | HG.3f | understand and use vector notation; calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors; understand and use the commutative and associative properties of vector addition; solve simple geometrical problems in 2-D using vector methods |
|-------|---|-------|---|

## 4 Measures and construction (A01,A02 & A03)

Pupils should be taught to:

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

### Measures

|                |   |       |  |
|----------------|---|-------|--|
| FG.4a          | interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; know rough metric equivalents of pounds, feet, miles, pints and gallons; make sensible estimates of a range of measures in everyday settings | HG.4a | use angle measure; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; understand and use compound measures, including speed and density |
| FG.4b          | understand angle measure using the associated language  |       |  |
| FG.4c<br>HG.4a | understand and use compound measures, including speed and density   |       |  |

### Construction

|       |  |                |   |
|-------|--|----------------|---|
| FG.4d | measure and draw lines to the nearest millimetre, and angles to the nearest degree; draw triangles and other 2-D shapes using a ruler and protractor, given information about their side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct cubes, regular tetrahedra, square-based pyramids and other 3-D shapes from given information | FG.4d<br>HG.4b | draw approximate constructions of triangles and other 2-D shapes, using a ruler and protractor, given information about side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct specified cubes, regular tetrahedra, square-based pyramids and other 3-D shapes |
|-------|--|----------------|---|

| Foundation Tier |  | Higher Tier |   |
|-----------------|--|-------------|---|
| FG.4e           | use straight edge and compasses to do standard constructions, including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle | HG.4c       | use straight edge and compasses to do standard constructions including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle |

### Mensuration

|       |  |                         |  |
|-------|--|-------------------------|--|
| FG.4f | find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles | FG.4f<br>FG.4i<br>HG.4d | calculate perimeters and areas of shapes made from triangles and rectangles; find the surface area of simple shapes by using the formulae for the areas of triangles and rectangles; find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids; convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres; find circumferences of circles and areas enclosed by circles, recalling relevant formulae; calculate the lengths of arcs and the areas of sectors of circles |
| FG.4g | find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids  |                         |  |
| FG.4h | find circumferences of circles and areas enclosed by circles, recalling relevant formulae  |                         |  |
| FG.4i | convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres   |                         |  |

| Foundation Tier | Higher Tier |
|-----------------|-------------|
|-----------------|-------------|

**Loci**

|       |   |       |   |
|-------|---|-------|---|
| FG.4j | find loci, both by reasoning and by using ICT to produce shapes and paths | HG.4e | find loci, both by reasoning and by using ICT to produce shapes and paths |
|-------|---|-------|---|

## Other Issues

### 11

## Spiritual, Moral, Ethical, Social, Cultural and Other Issues

### 11.1 Spiritual, Moral, Ethical, Social, Cultural and Other Issues

Mathematics provides opportunities to promote:

- *spiritual development*, through explaining the underlying mathematical principles behind some of the natural forms and patterns in the world around us
- *moral development*, helping pupils recognise how logical reasoning can be used to consider the consequences of particular decisions and choices helping them learn the value of mathematical truth
- *social development*, through helping pupils work together productively on complex mathematical tasks and helping them see that the result is often better than could be achieved separately
- *cultural development*, through helping pupils appreciate that mathematical thought contributes to the development of our culture and is becoming increasingly central to our highly technological future, and through recognising that mathematicians from many cultures have contributed to the development of modern day mathematics.

### 11.2 European Dimension

AQA has taken account of the 1988 Resolution of the Council of the European Community in preparing this specification and associated specimen papers.

### 11.3 Environmental Issues

AQA has taken account of the 1988 Resolution of the Council of the European Community and the Report *Environmental Responsibility: An Agenda for Further and Higher Education* 1993 in preparing this specification and associated specimen papers.

### 11.4 Citizenship

A number of specification references, particularly those for Statistics, promote the skills of enquiry and communication. They also encourage the skill of participation and responsible action in the educational establishment and/or communication.

### 11.5 Avoidance of Bias

AQA has taken great care in the preparation of this specification and associated specimen papers to avoid bias of any kind.

11.6 Health and Safety

Aspects of the specification, particularly within Statistics provide opportunities to promote Health and Safety issues.

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11.7 ICT

- (a) Pupils should be given opportunities to apply and develop their ICT capability through the use of ICT tools to support their learning in mathematics.
- (b) Pupils should be given opportunities to support their work by being taught to:
- (i) find things out from a variety of sources, selecting and synthesising the information to meet their needs and developing an ability to question its accuracy, bias and plausibility
  - (ii) develop their ideas using ICT tools to amend and refine their work and enhance its quality and accuracy
  - (iii) exchange and share information, both directly and through electronic media
  - (iv) review, modify and evaluate their work, reflecting critically on its quality, as it progresses.
- 

11.8 Other Issues

Mathematics provides opportunities to promote:

- *thinking skills*, through developing pupils' problem-solving skills and deductive reasoning
  - *financial capability*, through applying mathematics to problems set in financial contexts
  - *enterprise and entrepreneurial skills*, through developing pupils' abilities to apply mathematics in science and technology, in economics and in risk assessment
  - *work related learning*, through developing pupils' abilities to use and apply mathematics in workplace situations and in solving real-life problems.
-

# Awarding and Reporting

## 12

### Grading, Shelf-life and Re-sits

#### 12.1 Qualification Titles

The qualifications based on this pilot specification have the following titles:

AQA General Certificate of Secondary Education in Mathematics

#### 12.2 Grading System

The GCSE qualification will be graded on an 8 point grade Scale A\*, A, B, C, D, E, F, G. Candidates who fail to reach the minimum standard for grade G will be recorded as U (unclassified) and will not receive a qualification certificate.

Units 2 and 3 are offered at two tiers of entry, Foundation tier and Higher tier. For each unit, grades C–G are available for candidates entered for the Foundation tier and grades A\*–D for candidates entered for the Higher tier. There is a safety net for candidates entered for the Higher tier, where an allowed grade E will be awarded for the unit where candidates just fail to achieve grade D.

Candidates may enter Units 2 and 3 at different tiers. The final grade awarded will be dependent only on the total Uniform marks gained (see below).

#### 12.3 Subject Award

Candidates should be entered for the overall GCSE subject award at or after the series in which they complete the two units. This will lead to the aggregation of Units 2, 3 and award of a subject grade.

**12.4 The Determination of Candidates' Final Grades**

For Units 2 and 3, candidates' results will be reported on a *Uniform Mark Scale* which is related to grades as follows

**Unit 2 (Maximum Uniform Mark = 100)**

| Mark Range | Grade |
|------------|-------|
| 90 – 100   | A*    |
| 80 – 89    | A     |
| 70 – 79    | B     |
| 60 – 69    | C     |
| 50 – 59    | D     |
| 40 – 49    | E     |
| 30 – 39    | F     |
| 20 – 29    | G     |
| 0 – 19     | U     |

**Unit 3 (Maximum Uniform Mark = 200)**

| Mark Range | Grade |
|------------|-------|
| 180 – 200  | A*    |
| 160 – 179  | A     |
| 140 – 159  | B     |
| 120 – 139  | C     |
| 100 – 119  | D     |
| 80 – 99    | E     |
| 60 – 79    | F     |
| 40 – 59    | G     |
| 0 – 39     | U     |

A candidate's uniform mark is calculated from his/her raw mark for the unit by using the grade boundaries set by the Awarding Committee. For example, a candidate who achieves the minimum raw mark required for grade B on Unit 2 receives a uniform mark of 70.

A candidate cannot obtain a uniform mark corresponding to a grade which is above the range for the tier. For example, on Unit 2, a candidate entered for the Foundation tier (grade range C – G) cannot obtain a uniform mark higher than 69, even if he/she achieves the maximum (raw) marks for the paper.

On individual units there is a small ‘safety net’ for candidates who fail to reach the minimum mark required for the lowest grade available in the tier.

For example, on Unit 3, a candidate entered for the Higher tier (grade range A\* – D) who just fails to reach the standard required for grade D does not obtain zero uniform marks. However, centres should note that such a candidate will normally be awarded fewer uniform marks than a Foundation tier candidate who reaches the same standard.

A candidate’s overall uniform mark is obtained by adding together the uniform marks for the two units. This overall mark is then converted to a grade by means of the following correspondence.

**Overall (Maximum Uniform Mark = 300)**

| Mark Range | Grade |
|------------|-------|
| 270 – 300  | A*    |
| 240 – 269  | A     |
| 210 – 239  | B     |
| 180 – 209  | C     |
| 150 – 179  | D     |
| 120 – 149  | E     |
| 90 – 119   | F     |
| 60 – 89    | G     |

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**12.5 Shelf-life of Unit Results**

The shelf-life of individual unit results, prior to the award of the qualification, is limited only by the shelf-life of the specification.

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**12.6 Re-taking Units and Carrying Forward of Unit Results**

Each unit may be re-taken once before certification of the qualification. The better result for each unit will count towards the final award.

Candidates who wish to re-take the qualification after first certification may re-use results from any unit.

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**12.7 Awarding and Reporting**

The procedures for Awarding Grades and Reporting Results to centres comply with the GCSE Code of Practice issued by the Regulatory Authorities.

## Appendices

### A

## Functional Mathematics

### A.1 Rationale

This specification makes reference to the stand-alone Functional Mathematics at Level 2. This qualification will be reported and awarded separately and the marks gained will not contribute to the GCSE grade awarded. However, for the purposes of gathering robust pilot data, it is expected that candidates entering the GCSE Mathematics pilot will also sit the Functional mathematics assessment.

In Functional Mathematics, candidates reaching the required standard will be awarded a Level 2 qualification. Candidates who do not meet the standards for Level 2 may be awarded Level 1. Candidates who do not meet the standards for Level 1 will be reported as Unclassified. At present, it is intended that a pass in Functional Mathematics will be required before a grade C or better can be awarded at GCSE when functional skills become a national requirement. This pilot specification will **not** impose this hurdle, however candidates are expected to sit Functional Mathematics in order for the effects of such a hurdle to be rigorously modelled.

Functional Mathematics, as defined in *The functional skills standards: mathematics*, requires learners to have the skills and confidence to apply, combine and adapt their mathematical knowledge to new situations in their life and work. It has been described as applying straightforward mathematics in complex contexts.

In the standards, it is envisaged that differentiation is achieved not just through the technical demand of the mathematics required but also through the complexity and familiarity of the context in which the mathematics is presented, and the independence shown in tackling problems.

### A.2 Process Skills and Content

The Functional Mathematics standard at Level 2 has as the underpinning basis the following process skills.

Representing: Making sense of situations and representing them.

Analysing: Processing and using the mathematics

Interpreting: Interpreting and communicating the results of the analysis.

Further details can be found in the *Functional skills standards: mathematics*. At Level 2 it is expected that the learner can

- understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations
- identify the situation or problem and the mathematical methods needed to tackle it
- select and apply a range of skills to find solutions
- use appropriate checking procedures and evaluate their effectiveness at each stage
- interpret and communicate solutions to practical problems in familiar and unfamiliar contexts and situations
- draw conclusions and provide mathematical justifications

The content and skills required are equivalent to national curriculum mathematics levels 1–6.

Specifically, the *Functional skills standards: mathematics* at Level 2 require that the learner can:

- understand and use positive and negative numbers of any size in practical contexts
- carry out calculations with numbers of any size in practical contexts
- understand, use and calculate ratio and proportion, including problems involving scale
- understand and use equivalences between fractions, decimals and percentages
- add and subtract fractions; add, subtract, multiply and divide decimals to a given number of decimal places
- understand and use simple equations and manipulate simple formulae involving one or two step operations
- recognise and use 2-D representations of 3-D objects
- find area, perimeter and volume of common shapes
- use, convert and calculate using metric and, where appropriate, imperial measures
- collect and represent discrete and continuous data
- use and interpret for discrete and continuous data, statistical measures, tables and diagrams
- use statistical methods to investigate situations
- use a numerical scale from 0 to 1 to express and compare probabilities

### A.3 Assessment Structure

It is in the nature of the subject that the process skills from the standards will be assessed across both GCSE and the Functional Skills Certificate but achievement in Functional Mathematics as a qualification is assessed wholly within the Functional Mathematics unit.

The assessment is in two parts, Paper 1 and Paper 2.

Paper 1 is a competency test assessing the content above in a straightforward manner with little or no context. It consists of 30 single-mark questions in a question paper of 40 minutes duration. The use of a calculator is **not** permitted in Paper 1.

Paper 2 is a functionality test assessing the content and process skills with questions placed in contexts which may be unfamiliar and may be drawn from everyday life, the workplace or educational settings. It consists of 2 questions worth a total of 30 marks where the context is presented in advance on pre-released data sheets, and 3 or 4 questions also worth a total of 30 marks where the contexts are less complex and presented as part of the examination. The question paper is of 75 minutes duration. The use of a calculator is expected in Paper 2.

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### A.4 Pre-released Data Sheets

Pre-released data sheets will be sent to centres about 4 working weeks before the timetabled examination. Candidates will be able to work with their teachers in familiarising themselves with the contexts, clarifying any specialised vocabulary and considering the possible mathematics in the situation presented. Teachers will need to provide reasonable class time for this to happen. This should be at least one lesson but there is no upper limit on the time candidates can spend considering the data sheets.

The pre-released data sheets cannot be taken into the examination room. A clean copy of the data along with any further data required for the other questions will be issued to all candidates at the start of the examination. This sheet will not be collected in and marked so candidates should ensure that all working is presented in the question paper/answer book.

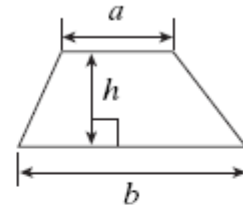
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### A.5 Further Support

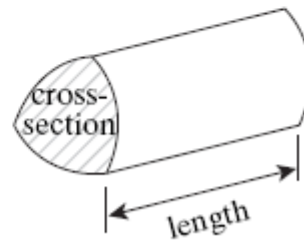
Support for the preparation of candidates for this new aspect of the specification will be provided by AQA and its partners.

**B****Formulae Sheets (Unit 3)****Foundation Tier**

**Area of trapezium** =  $\frac{1}{2}(a+b)h$

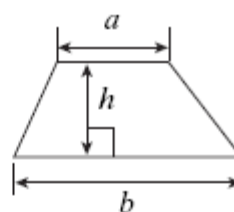


**Volume of prism** = area of cross-section  $\times$  length

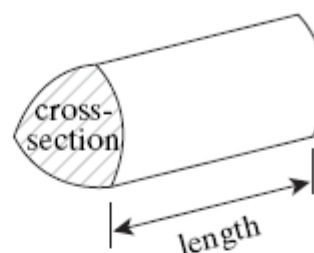


# Higher Tier

**Area of trapezium** =  $\frac{1}{2}(a+b)h$

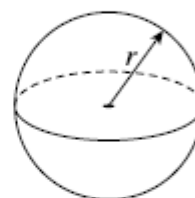


**Volume of prism** = area of cross-section  $\times$  length



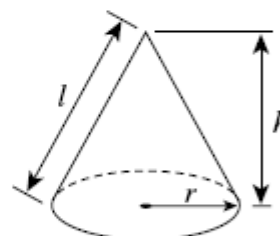
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4 \pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$

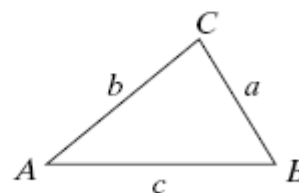


**In any triangle ABC**

**Area of triangle** =  $\frac{1}{2} ab \sin C$

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$



## The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

## C

## Overlaps with other Qualifications

The Functional Mathematics assessment at Level 2 which is referred to in this specification is identical to the stand-alone Functional Mathematics assessment at Level 2 detailed in AQA's Functional Mathematics pilot specification.

The content of this specification is identical, though differently structured, to that of AQA GCSE Mathematics Specifications A and B. The content is also identical to that of AQA Pilot GCSE in Additional Mathematics though the emphasis of the assessment is very different.

There is some overlap between GCSE Mathematics and GCSE Statistics.

There is some overlap of skills and content between GCSE Mathematics and Free Standing Mathematics Qualifications (FSMQs). In some post-16 centres candidates on the different courses may be grouped together.

Further information about the links between these subjects can be obtained from AQA.