Helping students to think about mathematics

1 Extending questions

In a typical maths exercise on proportional reasoning all problems are similar.

For example

Bill buys eight bottles of cola for £4.00.

What is the cost of each bottle?

To answer this routine question a student does not have to pick an appropriate mathematical model and little thinking is involved.

But what about

It takes 40 minutes to bake 8 potatoes.

How long does it take to bake one potato?

or If you buy ten bottles you get a 5% discount.

Is buying six bottles a week a good idea?

*Problems like this might force students to **think**!*

Students need to experience mathematical situations where they are forced to think for themselves.

Here is another situation involving proportional reasoning.

It takes 1.85 metres of denim to make a pair of jeans.

Denim costs £4.50 per metre.

The obvious question is

“How much does the denim cost to make one pair of Jeans?”

Again, to answer this routine question a student does not have to pick an appropriate mathematical model and little thinking is involved.

But what about

What happens if the price of denim increases by 10%?

or What if the cost of Denim is exclusive of VAT?

or A Tailor regularly makes Jeans for her customers.

She gets these discounts on the cost of Denim from her suppliers.

10% if she spends more than £50

15% if she spends more than £100

How many pairs of Jeans should she make at a time?

or Find some other contexts (and questions) where you might find similar information (and choose similar methods) to solve a problem.

For example

It takes 1.6 kg of flour to make 50 bread rolls.

Flour costs £1.95 per kilogram.

…

Proportional reasoning is a rich source of functional questions. Make sure students experience problem solving involving proportional reasoning.

2 Types of questions

Ask questions without an exact answer.

For example

How many chips have you eaten in your life?

How many seconds have you been alive?

How many hours did it rain last year?

What is the surface area of a banana?

How many leaves in an Oak tree?

The purpose of these questions is to get students to think about the issues involved in finding a solution (including the fact that any answer can only at best be an estimate).

For example

To estimate the number of chips eaten in your life some issues might be:

- The average number of chips you eat, say, in a week at the moment?
- How this might have increased as you have grown older?
- At what age you started eating chips?
- How to make the estimates?
- The assumptions that could be made?
- What is an acceptable range of answers?

Decisions about these issues can lead to a mathematical model of the situation from which an estimate can be worked out.

Ask open questions

For example

What set of data has a mean of 5 and a median of 6?

What shape has an area of 60 cm^2?

What can you tell me about the number 4.6?

What can you tell me about a rectangle?

What can you tell me about a quadrilateral?

What can you tell me about a triangle?

What equation has the solution $x = 5$?

What simultaneous equations have the solution $x = 5$ and $y = 0.5$?

What are the dimensions of a cone with a volume of 200 cm^3?

What is the surface area of a cone with a volume of 200 cm^3?

In attempting to answer open questions like these students have to think about the mathematics they know and the methods they can use.

Encourage them to develop their initial answers using a wider range of mathematics making as many choices as possible for themselves.

For example:

When asking what shape has an area of 60 cm^2 many students might restrict themselves to rectangles. Simple prompts like “Are there other shapes you could try?” extends this work into potentially fruitful areas.

This sort of activity is a good preparation for AO2 questions where students have to “choose and use”.

3 Algebra as a problem solving tool

Many problem solving questions can be solved by setting up equations.

In an AO2 question this would ***not*** *be prompted – students would be expected to choose to use an equation for themselves.*

For example

Tim has some marbles and some boxes.
When he puts 20 marbles in each box, there are 10 marbles left over.
When he puts 22 marbles in each box, there are 2 marbles left over.
How many marbles does Tim have?

Let there be $\boldsymbol{m}$ marbles and $\boldsymbol{b}$ boxes

$\boldsymbol{m} = 20\boldsymbol{b} + 10$

$\boldsymbol{m} = 22\boldsymbol{b} + 2$

$22\boldsymbol{b} + 2 = 20\boldsymbol{b} + 10$

$2\boldsymbol{b} = 8$

$\boldsymbol{b} = 4$

$\boldsymbol{m} = 20 \times 4 + 10 = 90$

Check

$\boldsymbol{m} = 22 \times 4 + 2 = 90$

There are other ways of solving this particularly problem (most students would use Trial and Improvement) but using an algebraic approach to problems like these often provides an efficient solution.

For example

The sum of two numbers is 10

The difference between the two numbers is 2.72

What are the numbers?

Using T & I in this example could be very inefficient! So …

Let the numbers be x and y …
or
Let the larger number be x …

4 Justifying decisions

*Ask if a statement sometimes **true**, **always true** or **never true**.*

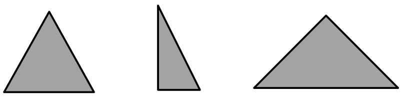
For example

A trapezium does not have a line of symmetry
Doubling each number in a set of data doubles the mean
Multiplying by an odd number gives an odd number
When you subtract you always get a smaller answer
Squaring a number always makes it bigger
A triangle has three acute angles
A triangle has two obtuse angles
A quadrilateral has three right angles

*Make each "item" the **odd one out** for as many different reasons as you can.*

(a) 32 36 45

(b)



(c) (6 5 6 12 6) (6 6 6 14) (12 5 7 10 7 7)

(d)

x	Frequency		
	Set A	**Set B**	**Set C**
0	1	1	0
1	1	2	6
2	7	7	2
3	6	6	10
4	0	4	1

(e)

Key 3 | 2 represents 32

Set **A**

0	5 7 9
1	0 1 3
2	2 6 6 6
3	2 5 7
4	0 5
5	

Set **B**

0	
1	5 6 7
2	2 6 6 6
3	2 5 7 9
4	0 1
5	4 5

Set **C**

0	5 7 9
1	0 1 3 4
2	2 6 6 6 8 9
3	2 5 7 9
4	0 1
5	5

*Sort out these statements into those you **agree** with and those you **disagree** with.*

${}^{2}/_{3}$ is bigger than ${}^{3}/_{5}$	$3 + 2 \times 4 = 20$
0.3 is the same as 30%	$2.5 \times 10 = 2.50$
$0.2 \times 0.4 = 0.8$	${}^{1}/_{2} + {}^{1}/_{3} = {}^{5}/_{6}$

*In each of these activities the thinking starts with the attempt to **justify** the decision so this is essential.*

5 Managing information

*An activity like this gives students develops experience in **classifying**, **ordering** and **sorting** often found in problem solving.*

Give students a set of cards with information about second-hand cars …

Category (Super-mini, Small family, …)
Body Style (Hatchback, Saloon, Estate, …)
Cost, age, mileage, …
Miles per gallon, CO2 emissions, …
Other factors such as number of seats, boot space …

You could ask students to

- Sort the cards according to criteria of their choice
- Find cars with specified criteria (one or more conditions)
- Find a car suitable for a given purpose
- etc ...

Experiencing situations where there is too much information or the wrong information is a good exercise.

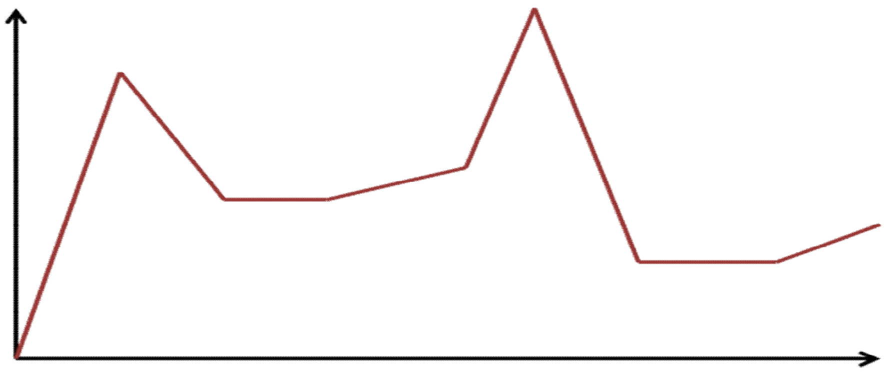
For example

What problem (or problems) can be solved using the information?
Is there any information missing?

- The bus stop is ½ a mile from my house
- The speed limit is 30 mph
- It takes me 24 minutes to walk
- I can walk at 5 mph
- The car never goes faster than the speed limit
- Buses come every 15 minutes
- The bus is number 32
- I hate cycling when its windy
- The car goes faster than the bus
- Going in the car takes half the time it takes me to cycle
- The bus journey usually takes 9 minutes
- The bus fare is 50 pence
- Each day I cycle, get the bus, walk or get a lift to school
- I can cycle at 12 mph
- Usually I wait at the bus stop for 5 minutes
- There is a bus stop outside school

6 Create a story from a graph or chart with no labels on the axes.

For example



7 Look for the maths in a situation

For example

What mathematical questions could you ask about …

- Booking a client for treatment in a beauty salon
- A shopping trip to buy some new clothes
- A summary of last season's results for two rival Premier League football teams
- A baked bean tin

8 What is the same? What is different?

For example

Compare these diagrams.

What is the same?
What is different?