

AQA Maths 2010 Ready Conferences, Autumn 2009

Seminar: New techniques for teaching Functional Maths and problem solving

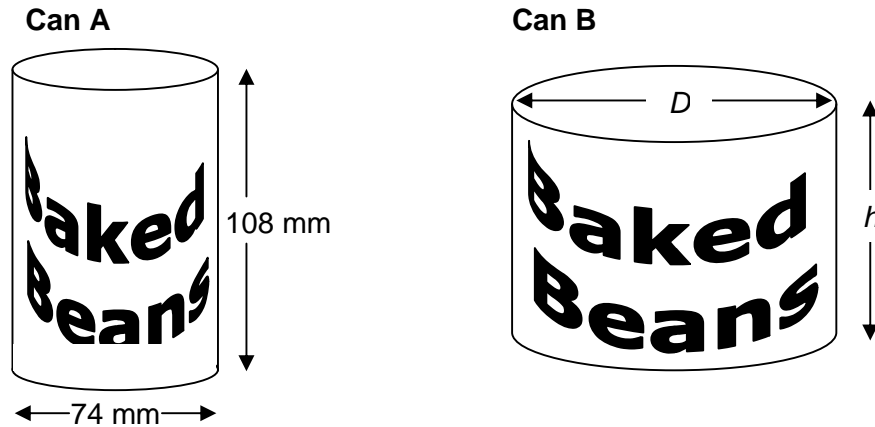
**Functional Elements
and
Problem Solving**

GCSE Questions and Activities

Questions that assess functional elements

Higher tier

- 1 The diagrams show two different cylindrical cans of Baked Beans.



Not drawn accurately

The diameter of Can A is 74 mm.

The height of Can A is 108 mm.

- (a) (i) Work out the volume of Can A.

Grade C AO1

- (a) (ii) Work out the total surface area of Can A.

Grade B AO1

- (b) Can A and Can B have equal volume and are made of metal of the same thickness.

The diameter, D , of Can B equals its height, h .

Compare the amount of metal used to make Can A with the amount of metal used to make Can B.

Grade A AO3

- N2 Tom and Jerry predict the finishing positions of all eight runners in a race. The table shows their predictions and the actual positions of the runners.

Actual position	1	2	3	4	5	6	7	8
Tom's predictions	3	2	6	1	4	5	7	8
Jerry's predictions	2	1	4	3	5	7	8	6

Overall, whose predicted positions are closest to the actual positions?

Give evidence to support your answer.

Grade C AO3

- 3 A company provides a 2 year extended guarantee for 3 kw electric kettles and 2 kw electric kettles.

This table provides some details of these guarantees.

	3 kw	2 kw
Number guaranteed	72 674	109 263
Average repair cost	£16.20	£14.70
Replacement cost	£28.65	£23.49
Probability of a repair	0.0083	0.0059
Probability of replacement	0.0036	0.0017

Work out the likely cost to the company of guaranteeing these kettles.

- N4 In 2009 a market gardener employs 6 workers to pick strawberries.

The workers pick the strawberries in 10 days.

In 2010 there is a 35% increase in the number of strawberries.

The market gardener decides to increase the number of workers by 50%.

How many days does it take to pick the strawberries in 2010?

Assume that the workers in both years pick strawberries at the same rate.

You **must** show your working.

Grade B AO2

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- 5 After exercise you can work out your fitness index, F , using this formula.

$$F = \frac{50T}{a + b + c}$$

You need to know

Your exercise time in seconds (T)

The number of your pulse beats in three 30 second intervals after you have stopped exercising (a , b and c)

Your fitness grade can be worked out from your fitness index, F , using this table.

Fitness index, F	< 50	50 to 60	60 to 70	70 to 80	80 to 90	≥ 90
Fitness grade	Very poor	Poor	Fair	Good	Excellent	Superb

Terry exercises for 210 seconds.

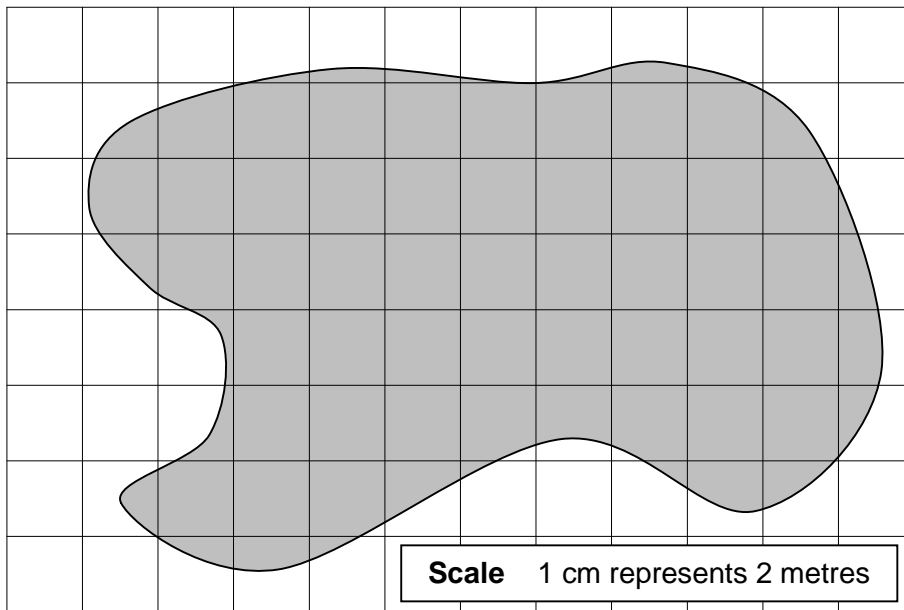
When he counts his pulse beats he obtains $a = 70$, $b = 55$ and $c = 45$.

What is Terry's fitness grade?

You **must** show your working.

Grade C AO2

- 6 The diagram shows the plan view of a landfill site on a centimetre grid. A landscape gardener is going to cover the site with turf (grass).



The table shows the cost of turf for different areas.

Area of turf (m ²)	Cost per square metre
40 – 59	£2.83
60 – 130	£2.33
131 – 240	£2.03
241 – 480	£1.78
481 – 640	£1.53
641 – 960	£1.40
961 – 1440	£1.23

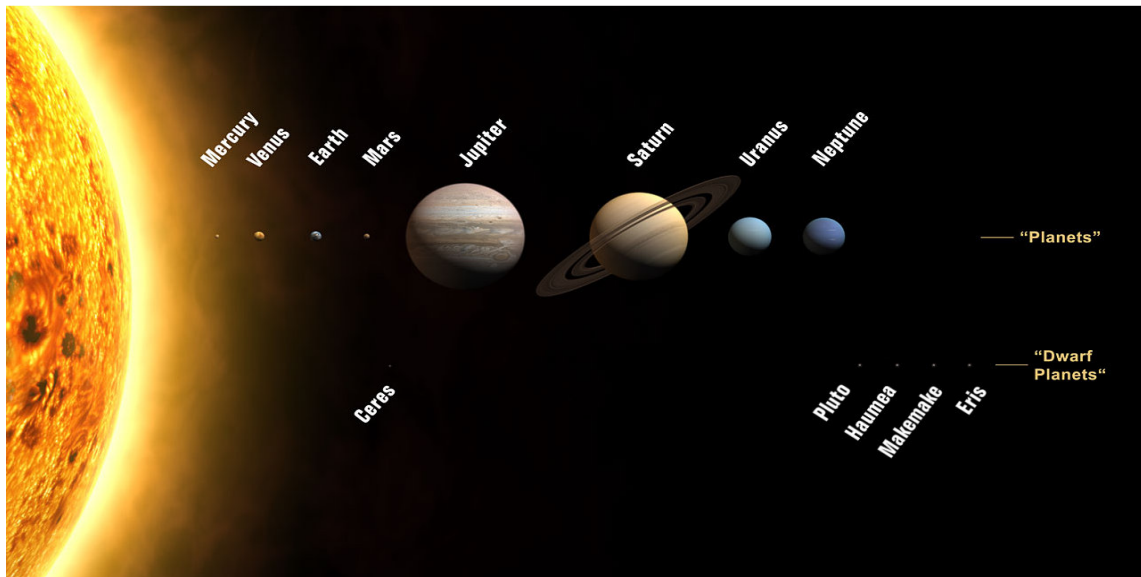
The landscape gardener must buy enough turf to cover the landfill site.

Work out how much he has to pay.

You **must** show your working.

Grade B AO3

- 7 The curator of a museum designs a scale model showing the eight major planets of the solar system.
The model is to hang from the ceiling of the museum's entrance hall.



The curator decides to make the model using the scales shown in this table.

Measurement	Scale
Distances between centres of planets	1 metre represents 4×10^{11} metres
Diameters of planets	1 metre represents 4×10^8 metres

- (a) The sun is in the centre of the model.
It has a diameter of 1.392×10^9 metres.
Work out the diameter of the sun in the model?
Grade A AO2
- (b) The entrance hall of the museum is circular with a diameter of 25 metres.
The furthest planet, Neptune, is approximately 30 Astronomical Units (30 AU) from the sun.
 $1 \text{ AU} = 1.496 \times 10^{11}$ metres (the distance from the Earth to the sun).
Will the model fit in the entrance hall?
Give evidence to justify your answer.
Grade A AO3
- (c) Eris is the largest of the dwarf planets.
Its distance from the sun is 96.7 AU.
What scale for distances would the museum have to use to make Eris part of the model?
Grade A AO3

Foundation and Higher

- 1 You can find out if your weight is healthy by working out your Body Mass Index (BMI).

Your BMI is worked out from your weight (W , kilogrammes) and height (h , metres) using this formula

$$\text{BMI} = \frac{W}{h^2}$$

You can use your BMI to work out whether you are classified as underweight, healthy, overweight or obese from this table.

Classification	BMI range
Underweight	Less than 18.5
Healthy	18.5 to 25
Overweight	25 to 30
Obese	Over 30

Tom's height is 1.63 metres.

His weight is 72.8 kilograms.

How much weight does Tom need to lose to be classified as Healthy?

Grade C AO3

- N2 Twenty miles per hour speed limit signs are put up on a housing estate.

The stem-and-leaf diagrams show the speeds of 15 cars both before and after putting up the signs.

Before	After
1 9 9	1 6 8 8 9 9 9
2 4 6 9 9	2 0 2 3 4 6
3 0 0 2 6 6	3 2 5
4 0 1 2	4 2
5 2	5 1

Key: 3 | 5 means 35mph

Compare the speeds of the cars before and after putting up the signs.

Grade D AO3

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- 3 The total length of all the fish that can live safely in a fish tank can be estimated using this formula

$$T = \frac{9VL}{10\,000}$$

T is the total length of the fish in the tank in centimetres

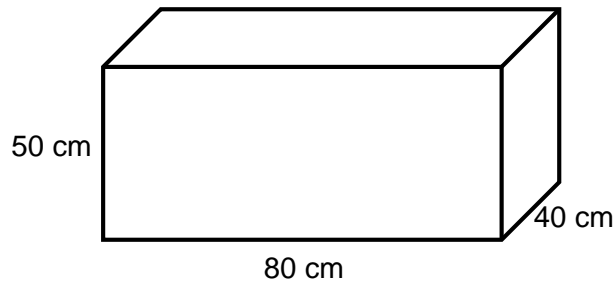
V is the volume of the tank in cubic centimetres

L depends on the type of filter in the tank.

Values of L , for different types of filter are shown in the table.

Type of Filter	L
Under-gravel	1.0
Internal filter	1.2
Oversized internal filter	1.4
External filter	1.7
Oversized external filter	2.0

Matthew buys this fish tank.



Not drawn accurately

The total length of fish that Matthew wants to live in this tank is 225 cm.

What type of filter should he buy so that they can do so safely?

You **must** show working to justify your answer.

Grade C AO3

Foundation tier

- N1 Sam and Matthew go to the cinema.
They each pay £5.75 to get in.

Large Popcorn	£4.20
Medium Popcorn	£3.00
Large drink	£2.80
Medium drink	£2.20
Small drink	£1.80
Nachos	£5.50

They decide to share a Large Popcorn.
Sam buys a Large drink and Matthew buys a Small drink.
How much does their trip to the cinema cost?

Grade F AO2

- N2 The Centre for Life is a museum in Newcastle upon Tyne.

Centre for Life	
<i>Admission charges</i>	
Adult	£6.95
Child (under 4)	Free
Child (5 to 16 years)	£4.50
Family*	£19.95
* Family	2 adults and 2 children or 1 adult and 3 children

- (a) Mr. Smith and his 8 year old son visit the Centre for Life.
Work out the total Admission charge they have to pay.

Grade G AO1

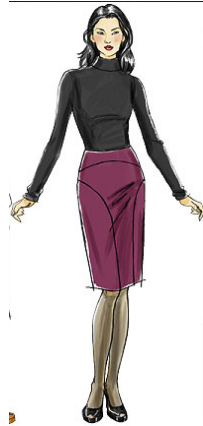
- (b) Mr and Mrs Jones and their two children aged 8 and 11 visit the Centre for Life.
How much do they save by buying a Family ticket rather than a separate ticket for each person?

Grade F AO2

- 3 Ben earns £24 000 per year.
He is paid an equal amount each month.
He asks for a wage rise of £100 per month.
He is offered a 6% wage rise.
Compare the offer with what he asks for?

Grade E AO2

N4 A dress maker has orders to make three skirts.



One of the skirts is size 8, one size 10 and one size 14.
The dress maker needs to buy the material to make the skirts.
The table shows the length of material needed.

Size	8	10	12	14	16	18
Length of fabric	1.9 m	1.9 m	1.9 m	2.2 m	2.2 m	2.2 m

The material costs £5.50 per metre.

How much does the dress maker have to pay for the material to make these skirts?

Grade F AO2

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N5 Tony works 21 hours altogether from Monday to Friday.
He works 5 hours on Saturday.
His pay is £6.50 per hour for Monday to Friday.
On Saturday he is paid an extra 30% per hour.
How much does he earn for the whole week?

Grade E AO2

N6 John has these coins.



(a) Write down all the different ways that John can use these coins to pay for a pen that costs 22 pence.

AO2

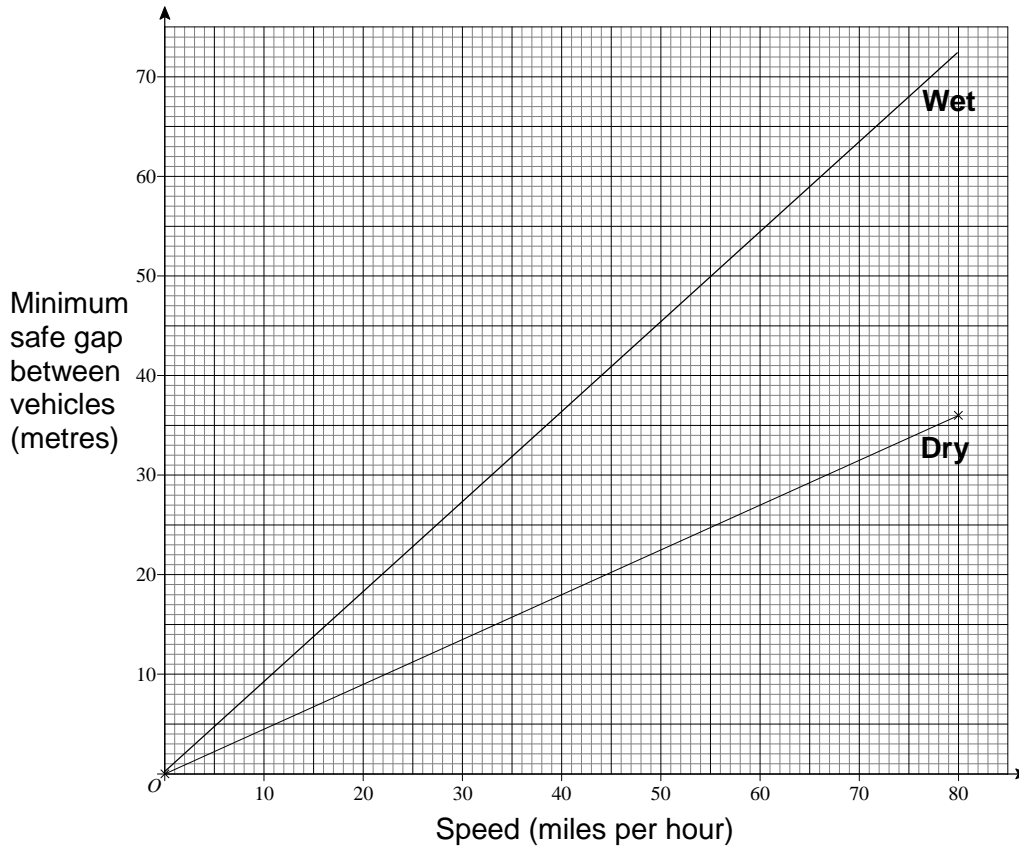
(b) Make a list of all the different amounts of money can be made with exactly four of these coins.

AO3

N7 Motorists should drive with a safe gap between their vehicle and the vehicle in front.

This graph shows the minimum safe gaps between vehicles at different speeds.

Different gaps are recommended for wet roads and dry roads.



Arnie is driving at 70 miles per hour on a dry road with the minimum safe gap between his car and the car in front.

It starts to rain heavily and both cars slow down to 50 miles per hour.

Arnie does **not** change the gap between his car and the car in front.

Is the gap still safe?

You **must** show clearly how you obtain your answer.

N8 Here are some instructions for cooking a turkey.

Cook for 20 minutes at 220°C Reduce the oven temperature to 170°C and cook for 50 minutes per kilogram
--

Davina is going to cook an 8 kilogram turkey.

She wants to take it out of the oven at 2:15 pm.

At what time must she start to cook it?

Functional Activities

Whole class group activities

Higher tier

- 1 A cylindrical Can contains 330 ml of pop.
 - (a)
 - (i) Work out possible dimensions for one of these cans.
You **must** show your method.
 - (ii) Work out the surface area of your can.
 - (iii) Compare your results with other members of your group.
Which dimensions give the smallest surface area?
 - (b) Work out the dimensions of a Can containing 330 ml that is made from the smallest amount of metal possible.
You can use a spreadsheet if you wish.

2 A trip to Lakeside Activity Centre

Lakeside Activity Centre

Cost arrangements for School Trips

Accommodation costs (per person per night).

	Mar/Apr	May/Jun	Jul/Aug	Sep/Oct
Adults	£33	£40	£38	£33
12 to 18 years	£28	£35	£33	£28
11 years and under	£27	£33	£31	£27

Cost of activities (per person, all ages)

	Mar/Apr	May/Jun	Jul/Aug	Sep/Oct
Full day	£23	£28	£27	£23
Half day	£18	£18	£17	£13
Evening	£5	£10	£10	£5

Special arrangements for school parties

There **must** be one member of staff for every 8 pupils.

For every 15 pupils, activities are free for **one** member of staff.

There is a discount of 12.5% on the total cost for 80 or more pupils.

A school organise a trip to Lakeside.

They take 100 year 9 pupils aged 13 and 14 and exactly the required number of staff.

This table shows the plans that the school make for the trip.

Day		Travel	Activities	Accommodation
Monday3 rd April	am	Travel to Lakeside	✗	✓
	pm		Half day	
	Evening		Evening	
Tuesday4 th April	am		Full day	✓
	pm			
	Evening		Evening	
Wednesday 3 rd April	am		Half day	✗
	pm	Travel back to school	✗	

(a) All pupils and members of staff do **all** of the planned activities.

Work out the total cost for accommodation and activities.

You **must** show your method.

(b) The school decides to pay for all travel costs and one-third of the accommodation and activity costs for staff and pupils.

All the remaining costs are shared equally among the pupils.

How much does each pupil have to pay to go on the trip?

Higher and Foundation

1 Terri sees this entry in an on-line catalogue.

PS3	£679.00
2-year guarantee	£65.00
30% discount for first purchases	

This table shows the different Options that Terri has for paying for the PS3.

Option	Deposit	Credit charge	Payment
A	25%	No charge	Pay the balance in equal amounts over 20 weeks
B	35%	5% of the balance	Pay the balance and the credit charge in equal amounts over 40 weeks
C	50%	10% of the balance	Pay the balance and the credit charge in equal amounts over 60 weeks

Terri buys the PS3 and the 2-year guarantee.
It is her first purchase so she gets 30% discount.

Compare the different ways she can pay.

Deposit	The initial amount you have to pay when you buy something expensive
Balance	What you owe after paying the deposit
Credit	A loan that covers the balance that is repaid over a period of time
Credit charge	What you are charged for credit.

When you are asked to **compare** two or more different things you need to write a short sentence using words like "more" or "less".

For example:

With Option A Terri pays £74.40 **less** deposit than with Option B.

In this Activity you need to compare:

- The deposits
- The credit charge
- The weekly payments

2 Stocking a fish tank

Data

The *total length* of fish that can live safely in a fish tank can be estimated from

- The *volume of water* in the tank
- The type of *filter*

Type of filter	Total length of fish in tank
Under-gravel	1.0 cm per litre of water in tank
Internal filter	1.2 cm per litre of water in tank
Oversized internal filter	1.4 cm per litre of water in tank
External filter	1.7 cm per litre of water in tank
Oversized external filter	2.0 cm per litre of water in tank

Some tropical fish

Fish	Fully grown length	Cost per fish
Euro Blue Ram	5 cm	£7.95
Rainbow Cichlid	8 cm	£9.95
Boesmani Rainbow	10 cm	£5.95
Neon Dwarf Rainbow	4 cm	£5.95
Peacock Gobi	10 cm	£5.75
Peppered Corydoras	5 cm	£2.95
Red Finned Shark	6 cm	£4.95
Siamese Flying Fox	15 cm	£2.50
Indian Glassfish	4 cm	£3.95
Panda Garra	10 cm	£12.00
White Cloud Minnow	4 cm	£1.25
Cherry Barb	3 cm	£2.45
Neon Gobi	3 cm	£5.75
Harlequin Lancer	8 cm	£12.95
Rainbow Garra Garra	8 cm	£11.95
Sailfin Glassfish	5 cm	£9.95
Humphead Glassfish	7 cm	£19.95

Example

The volume of water in a tank with an oversized internal filter is about 220 litres.

An estimate of the total length of fish that can live in the tank = $220 \times 1.4 = 308$ cm

The tank is stocked with 30 Peacock Gobi with a total length of $30 \times 10 = 300$ cm

Calculating the volume of water in a tank

- 1 Work out the total volume of the tank (Use m^3 , cm^3 or ml^3).
- 2 Convert to litres
(1 litre = $0.001 \text{ m}^3 = 1000 \text{ cm}^3 = 1\,000\,000 \text{ ml}^3$)
- 3 Deduct 10% for decor and equipment
- 4 Round to a sensible degree of accuracy

Example

A tank in the shape of a cuboid has dimensions 120 cm by 45 cm by 45 cm.

Total volume of tank $120 \times 45 \times 45 = 243\,000 \text{ cm}^3$

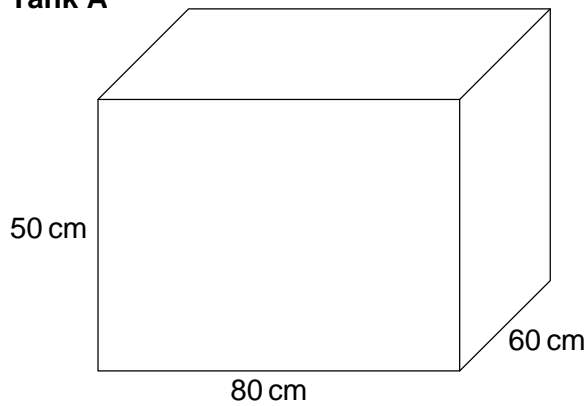
Converting to litres $243\,000 \div 1000 = 243 \text{ litres}$

Volume of water in tank $90\% \text{ of } 243 = 218.7 \text{ litres}$

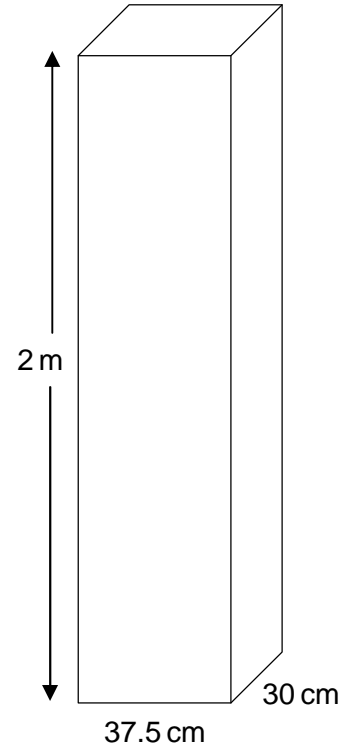
Volume of water for estimating the total length of fish in the tank = 220 litres

Two fish tanks

Tank A



Tank B



Closed questions

- (a) How much does it cost to buy 4 Harlequin Lancers, 5 Indian Glassfish and 3 Humphead Glassfish?
- (b) The total volume of a tank with an internal filter is 350 litres.
What is the total length of fish that can live safely in the tank?
- (c) Billy keeps fish in tank A fitted with an under-gravel filter.
He replaces the under-gravel filter with an internal filter.
What extra length of fish can live in the tank?
- (d) Sue wants to keep the following fish in tank B.
- | | |
|-----------------------|----------------------|
| 5 Neon Dwarf Rainbows | 10 Cherry Barbs |
| 10 Euro Blue Rams | 5 Red Finned Sharks |
| 8 Rainbow Garra Garra | 9 Peppered Corydorus |
- Can these fish live safely in the tank with an internal filter?
You **must** show your working.
- (e) Tank A has an oversized **internal** filter.
Tank B has an oversized **external** filter.
Compare the total length of fish that can safely live in each tank.

Open questions

- (f) You are planning to stock **tank A** using an external filter.
Write a list of the fish you could buy with a budget of £500.
Check that the fish can live in the tank safely.
- (g) Terry is building a cuboid-shaped fish tank to fit on a shelf 50 cm long and 30 cm deep. He plans to use an oversized external filter.
He wants the tank to hold these fish safely
- | | | |
|-----------------|---------------|----------------------|
| 6 Euro Blue Ram | 8 Cherry Barb | 4 Peppered Corydoras |
|-----------------|---------------|----------------------|
- (i) Terry uses the table below to decide on the thickness of glass he should use to make the tank.

Height of water (mm)	Thickness of glass (mm)
less than 150	4
between 150 and 180	6
more than 180	10

Work out the thickness of glass that Terry should use for his tank.

- (ii) Terry chooses the height of the glass for the tank so that the water level is 2 cm below the top of the tank.
He then works out the total length of edges that have to be sealed using a silicon sealant.
Work out the total length of the edges that need to be sealed.

2 Starting a Crèche

Data

Crèche Groups

- A crèche may include more than one group of children.
For children aged 2 years or over, the size of each group should not exceed 26 children.
For children under 2 years, the group size should not exceed 12 children.
- There should be one toilet and one wash hand basin for every 10 children over the age of 2 years.

Staffing Ratios

Age of children	Minimum adult : child ratio
Under 2 years	1 : 3
2 years	1 : 4
3 to 7 years	1 : 8

- There should always be a minimum of 2 adults on duty.
- Any care provided for children aged 8 – 14 should not adversely affect the care provided for children under 8 years old.

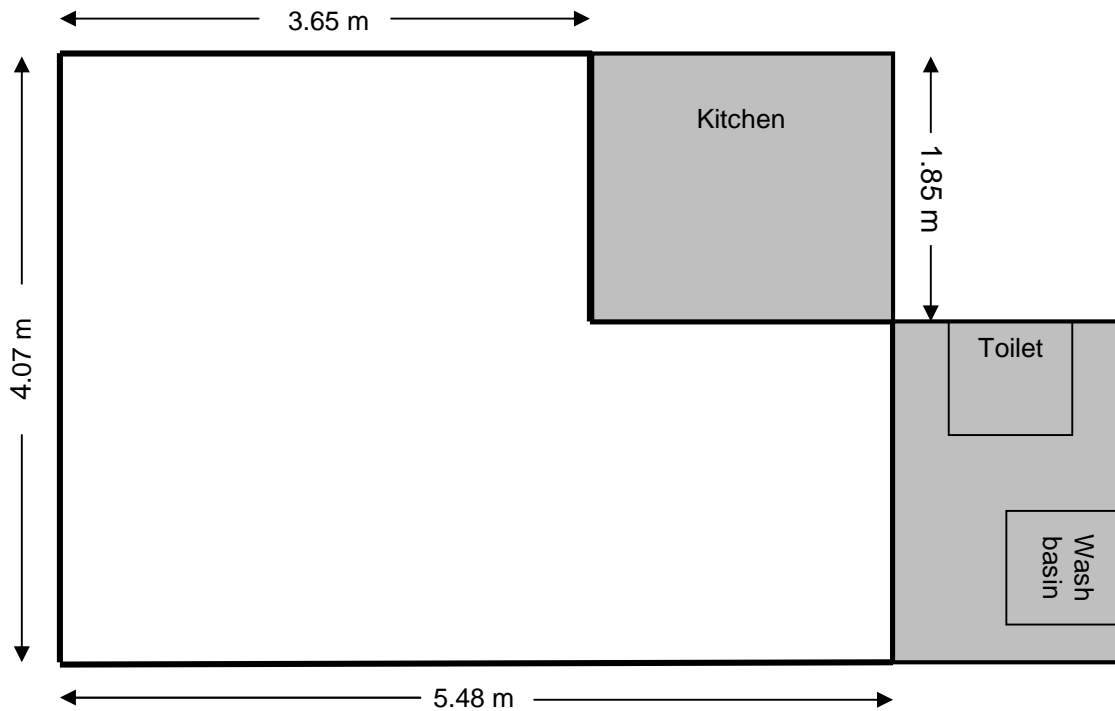
Space Standards

Age of children	Minimum space per child
Under 2 years	3.5 m ²
2 years	2.5 m ²
3 to 7 years	2.3 m ²

Staff and wages

Job	Salary
Manager	£370 per week
Nursery nurse	£280 per week
Nursery assistant	£6.50 per hour

Imagine you are planning to start a creche.
You are going to use the room shown below.



You want the creche to open for 10 hours a day from Monday to Friday.

- Give full details of **two** possible ways that you could run the creche.
- Write a list of any extra information that you might need.

3 Smoking kills

Many packets of cigarettes carry a “**smoking kills**” warning in large **black** print.

Here are some statistics from the Government White Paper “Smoking Kills” that support this.

- It is estimated that in the UK smoking kills more than 13 people an hour.
- Out of every thousand people who start smoking when they are teenagers and continue to smoke 20 a day, one will be murdered, six will die in road accidents and **about 250 will be killed before their time by smoking.**
- On average, each cigarette shortens a smoker's life by around 11 minutes.

Use this information to write some questions.

Work in a pair.

Try your questions out on another pair.

Do you get the same answers?

Foundation tier

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	Evening		Evening	
Tuesday ^{4th} April	am		Full day	✓
	pm			
	Evening		Evening	
Wednesday ^{3rd} April	am		Half day	*
	pm	Travel back to school	*	

All pupils and members of staff do **all** of the planned activities.

You have a holiday job at Lakeside and the manager asks you to complete the form below giving details of the costs for this trip

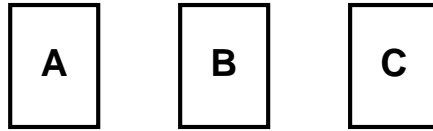
Copy and complete the form.

Numbers on trip	
<i>Adults</i>
11 to 18	100
Under 11	0
Number of Adults with free activities
Accommodation costs	
Adults	£.....
11 to 18	£.....
Under 11	£0.00
Activity costs	
Full day	£.....
Half day	£.....
Evening	£.....
Cost	
Accommodation + Activities	£.....
Discount	£.....
Total cost	£.....

Problem solving questions

Higher tier

- 1 A number is written on the other side of each of these cards.



The number on card **A** multiplied by the number on card **B** is 12.

The number on card **B** multiplied by the number on card **C** is 36.

The number on card **A** multiplied by the number on card **C** is 27.

Work out the number on card **A** multiplied by the number on card **B** multiplied by the number on card **C**.

Grade B AO3

- 2 Lizzy is given a new stamp album for her birthday.
Here are two different ways she could put her stamp collection into the new stamp album.

If she puts 10 stamps on a page there are 2 stamps left over on one page and 15 pages left in the stamp album.

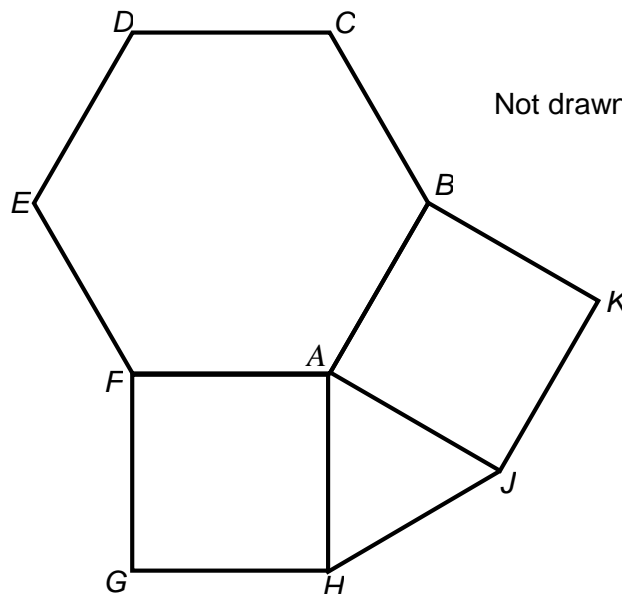
If she puts 12 stamps on a page there are 6 stamps left over on one page and 28 pages left in the stamp album.

How many stamps does Lizzy have in her collection?

How many pages are there in her new stamp album?

Grade A AO3

- N3 $ABCDEF$ is a regular hexagon.
 $AFGH$ and $AJKB$ are squares.



Prove that triangle AHJ is equilateral.

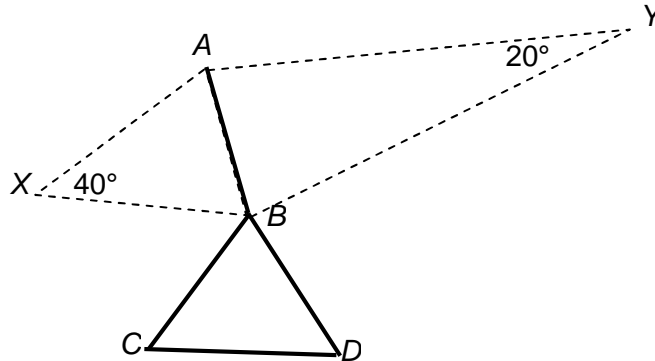
Grade A AO3

N4 AB and BC are two sides of a regular polygon with centre X .

Angle $AXB = 40^\circ$.

AB and BD are two sides of a regular polygon with centre Y .

Angle $AYB = 20^\circ$.



Prove that triangle CBD is equilateral.

Grade A AO3

5 Four friends Andy, Ben, Charlie and Della are playing a game. During the game they collect red and blue cards.

The table shows the numbers of cards each player collects during the game.

Player	Red cards	Blue cards
Andy	10	8
Ben	11	4
Charlie	13	11
Della	9	2

- (a) Each red card is worth r points.
Each blue card is worth b points.

Write an expression in terms of r and b for the total number of points won by Andy.

Grade D AO1

- (b) Altogether Andy wins 64 points and Ben wins 80 points.
The player with the greatest total number of points wins the game.
Which of the four players wins the game?
You **must** show your working.

Grade A AO3

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- 6 Each number in a sequence is three more than the number before it.
The sum of the first three terms of the sequence is 30.
Show that 301 is a term in the sequence?
Grade B AO3
- 7 A Farmer has some sheep.
80% of his sheep have lambs.
- 30% of the sheep who have lambs have one lamb
 - 40% of the sheep who have lambs have two lambs
 - The rest of the sheep who have lambs have 3 lambs
- A total of 240 lambs are born.
What is the total number of sheep and lambs that the Farmer has?
You **must** show your working.
Grade A AO3
- 8 The length of a rectangle is increased by 20%.
The width of the same rectangle is decreased by 20%.
Work out the percentage change in the area of the rectangle.
Grade A AO3
- N9 The n th term of a sequence is $T_n = 5n - 3$.
For example $T_4 = 17$
- (a) (i) Show that $T_1 + T_2 = 9$
Grade C AO1
- (a) (ii) Show that $T_{n+1} = 5n + 2$
Grade B AO2
- (a) (ii) Show that $T_{n+1} + T_{n+1}$ is a multiple of 10
Grade A AO2
- (b) In a different sequence, $T_n = 3n^2$
Show that $T_{n+1} - T_n = 6n + 3$
Grade A AO3
- 10 There are some red, white and blue counters in a box.
The ratio of the number of red counters to white counters is 3 : 5
The ratio of the number of white counters to blue counters is 2 : 5
What is the smallest possible number of counters in the box?
Grade C AO3

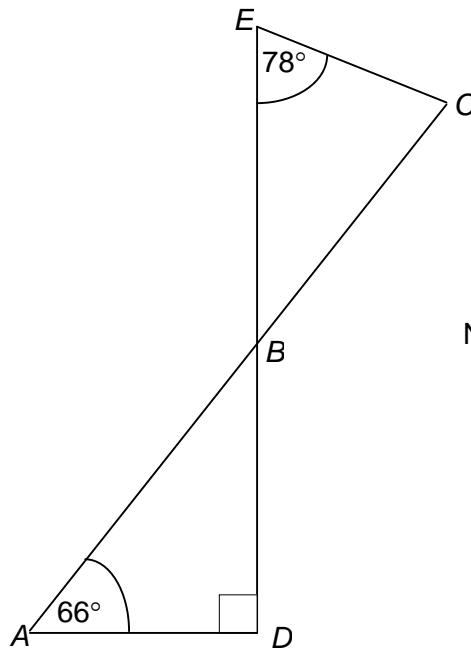
Foundation and Higher

N1 ABC and DBE are straight lines that intersect at B .

AD and DE are perpendicular.

Angle $BAD = 66^\circ$

Angle $BEC = 78^\circ$

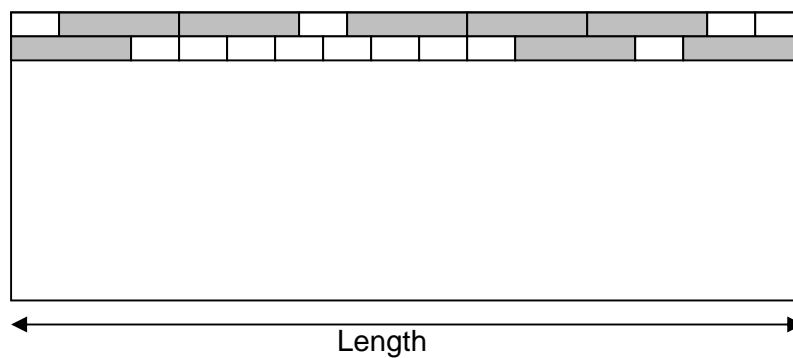


Not drawn accurately

Show that EB and CB are of equal length.

Grade D AO3

N2 Eric uses planks of wood of two different lengths to cover a floor in a room.



Not drawn accurately

The longer plank is 2 metres long.

What is the length of the room?

Grade C AO3

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- N3 Tom, Lee and Sam are trying to win some marbles in a game.
 Tom wins $\frac{2}{5}$ of the marbles.
 Lee and Sam win the remaining marbles in the ratio 7 : 5.
 What percentage of the marbles does Lee win?

Grade C AO3

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- N4 The n th term of sequence **A** is $2n + 3$
 The n th term of sequence **B** is $4n - 1$
 (a) Work out the first two terms of sequence **A**.

Grade D AO1

- (b) Work out the difference between the 20th term of sequence **A** and the 20th term of sequence **B**.

Grade D AO1

- (c) Terms in sequence **C** are made by finding the difference between equivalent terms in sequence **A** and sequence **B**.
 So the answer to part (b) is the 20th term in sequence **C**.
 Work out the n th term of sequence **C**.

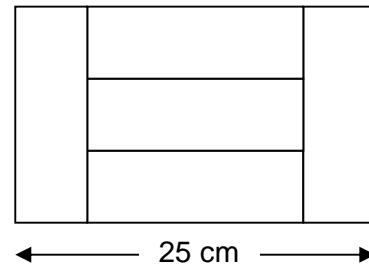
Grade C AO2

- (d) Show that 11 is a term in both sequence **A** and sequence **B** but **not** sequence **C**.

Grade C AO2

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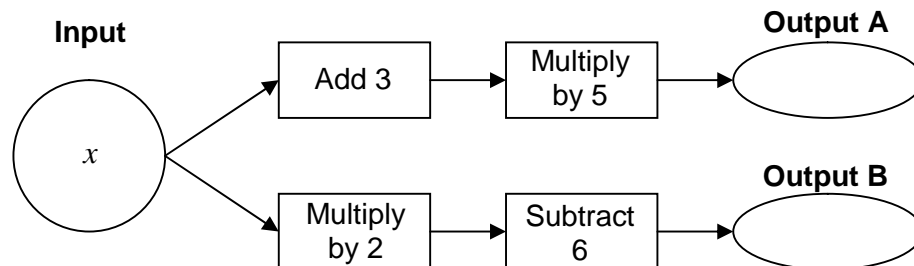
- N5 Five identical rectangles fit together as shown.



Work out the total area.

Grade C AO3

- N6 Here is a flow chart.



Work out the **input**, x , when **output A** equals **output B**.

Grade C AO2

Foundation tier

N1 You are given these numbers.

3.6 9.6 12.6 15.4 7.6 12.4 17.4 21.4

Group the numbers in pairs.

The sum of the two numbers in each pair must be the same.

Grade G AO2

N2 You can make a new number by swapping the positions of two digits in a number.

For example

You are given the number 23 187.

Swap 3 and 8 so that 23 187 → 28 137

Swap 2 and 7 so that 28 137 → 78 132

So 23 187 can be changed to 78 132 with **two** swaps.

(a) You are given the number 25 147.

(i) What is the largest number you can make using **one** swap.

Grade G AO2

(ii) What is the smallest number you can make using **two** swaps.

Grade F AO2

(b) You are given the number 3465

(i) The 4 and 3 are swapped.

Work out difference between the old number and the new number.

(ii) The 5 and the 4 are swapped and then the 4 and the 6 are swapped.

Work out the difference between the old and the new number.

Grade F AO2

(c) How many swaps does it take to change the number 1234 to the largest possible **even** number?

Show the numbers you make after each swap.

Grade F AO3

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N3 The point $(a, 2a)$ lies at intersection of the lines $x = 4$ and $y = b$.

Work out the value of b .

Grade E AO2

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N4 Tim, Stacey and Afzal think of a number and give each other clues about it.

(a) Here are the clues for Tim's number.

CLUE 1 My number is a multiple of 2

CLUE 2 My number is a multiple of 5

CLUE 3 My number is a three digit square number

Work out Tim's number.

Grade G AO3

(b) Here are the clues for Stacey's number.

CLUE 1 My number is a two digit factor of 48

CLUE 2 When I add the digits of my number I do **not** get a factor of 48

Work out Stacey's number.

Grade F AO3

(c) Here are the clues for Afzal's number.

CLUE 1 My number is a two digit square number that is also a cube number

Work out Afzal's number.

Grade E AO3

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N5 A container weighs 21 kg when full of water.

When it is a quarter full of water it weighs 9 kg.

What is the weight of the container when empty?

Grade E AO3

N6 The sum of two numbers is 50.

The difference between the two numbers is 10.

What are the numbers?

Grade F AO2

N7 A fence has five equally spaced posts.

Each post is 10 cm wide.

The length of the fence is 230 cm.

How wide is the gap between each post?

Grade E AO3

Problem Solving Activities

Whole class group activities

Higher tier

1 Saving aluminium

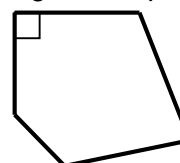
A cylindrical aluminium Can contains 330 ml of pop.

- (a) (i) Work out possible dimensions for one of these cans.
You **must** show your method.
- (ii) Work out the surface area of your can.
- (iii) Compare your results with other members of your group.
Which dimensions give the smallest surface area?
- (b) Work out the dimensions of a Can containing 330 ml that is made from the smallest amount of aluminium possible.
You can use a spreadsheet if you wish.

2 Right angles in a polygon

Work out the maximum possible number of interior right angles in a pentagon.
Show working to justify your answer.

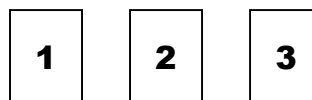
Investigate the maximum number of right angles in other polygons.



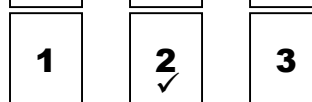
3 Choice or chance

An activity for two (*To find out more about this activity Google "Monty Hall"*)
You need three playing cards – one red and two black.

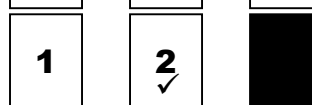
Player 1 shuffles the cards and deals them face down on the table.



Player 2 chooses one of the cards.
(For example: Card 2)



Player 1 looks at the two other cards and turns one that is **black** face up.



Player 1 can then choose to **stick** with their original card or **swap** it for the other card. Their objective is to pick the **red** card.

Should Player 1 stick or swap?

Record the outcome in a copy of this table.

Trial	Stick	Swap
1	Red	Black

This activity is probably best introduced with the teacher going through the routine at the front of the classroom with the class discussing to whether or not to swap.

Players 1 and 2 should discuss whether it is best to stick or swap and present the results of their discussion as a hypothesis. For example:

The red card is more likely to be picked if the card is swapped.

Repeat the trial a number of times to check your hypothesis.

Try and explain your results.

Higher and Foundation

- 1 You are given the starting number 67 234.
You can swap the positions of any two digits.
For example

Swapping the 1 and the 4 gives $67\ 234 \rightarrow 64\ 237$

A number made after one swap is called a “**one-swap**” number

A number made after two swaps is called a “**two-swap**” number
and so on ...

The table shows some examples.

Starting number	One-swap number	Two-swap number	Three-swap number
67 234	64 237	46 237	46 273
12 345	21 345	12 345	21 345

- (a) The starting number is 12 345.
(i) Show that 54 321 is a **two-swap** number.
(ii) Show that 41 253 is a **four-swap** number
- (b) The starting number is 12 345.
Work out all possible one-swap numbers?
- (c) The starting number is 1234.
Work out all the possible three-swap numbers?
Show your working.
- 2 **All** of the digits 3, 4 and 7 are used in a calculation **once only** to make a square number.
For example: $(7 - 4) \div 3 = 1$
- (a) Use these digits **once only** to make all the square numbers less than 50 apart from 4.
Write your calculations using the correct hierarchy of operations (BODMAS/BIDMAS).
- (b) You are now allowed to use as many of the digits 3, 4 and 7 as you like.
Try to make other square numbers using the smallest number of these digits as possible.

3. (a) Mr and Mrs Smith have seven children.
Five of the children are boys and two of them are girls.
The children buy chocolate eggs to give to each other.

Each girl gives each boy a red egg.
Each boy gives each girl a yellow egg.
Each girl gives each of the other girls a blue egg.
Each boy gives each of the other boys a green egg.

How many eggs of each colour do the children buy?
You **must** show your working.

- (b) Mr and Mrs Jones' children use the same rules for giving eggs.

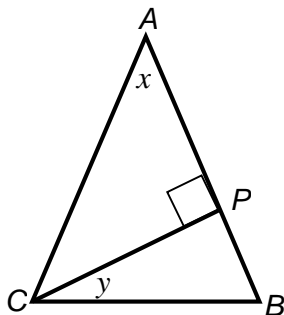
They buy
12 red eggs
12 yellow eggs
12 blue eggs
6 green eggs

How many boys and how many girls are in Mr and Mrs Jones' family?
You **must** show your working.

Extension

- (c) Look at part (b)
The problem can be solved with less information.
What information is **not** necessary?

- 4 ABC is an isosceles triangle.
 $AB = AC$
 P is a point on AB such that CP is perpendicular to AB .



Teacher's note

Simple activities like this can be used to illustrate the difference between a demonstration and a proof.

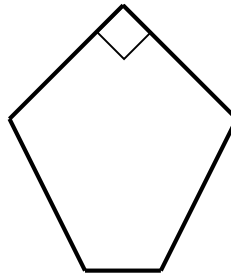
F tier students could be encouraged to demonstrate that $y = x \div 2$ by choosing values for x (or y) and then working out y (or x)

H tier students could be encouraged to go on to provide a formal proof for this.

Investigate the connection between the size of angle x and the size of angle y .
(AO3)

5 **Polygon codes**

Here is a polygon



It has **5** sides
1 interior right angle
0 pairs of opposite sides parallel
1 line of symmetry

The **code** for this polygon is **5 1 0 1**

(a) (i) The code for a square is **4 4 2 2**
 Explain why.

(ii) Here are some possible codes for other quadrilaterals.

4	0	0	0	4	0	2	2
4	1	0	0	4	0	0	1
4	1	0	1	4	0	2	0
4	0	1	0	4	4	2	2
4	1	1	0	4	2	1	0
4	0	1	1				

Draw and name each of these quadrilaterals explaining why one of them is impossible to draw.

Does the list contain the code for **all** the possible quadrilaterals that can be drawn?

Draw the **diagonals** of each of your quadrilaterals and describe how they intersect each other.

(b) Write down the codes for each of these triangles.

- (i) Scalene
- (ii) Isosceles
- (iii) Right angled
- (iv) Isosceles right angled
- (v) Equilateral

(c) Investigate the possible codes for pentagons, hexagons, ...