

GCE 2005

January Series



ASSESSMENT and
QUALIFICATIONS
ALLIANCE

Mark Scheme

Mathematics

MPC1

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous		
	incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

MPC1

Q	Solution	Marks	Total	Comments
1(a)(i)	Attempt at $\Delta y / \Delta x$ (used with numbers)	M1	2	Not x over y
	$= \frac{3}{12} = \frac{1}{4}$	A1		0.25 etc any correct equivalent
	(ii)	$y - 2 = m(x - 11)$ or $y + 1 = m(x + 1)$ $4y - x = -3$ etc leading to $x - 4y = 3$	M1 A1	2
(b)	Attempt to eliminate x or y $y = 1$ $x = 7$	M1 A1 A1	3	$17y = 17$ etc C is point (7,1)
Total			7	
2(a)	$\frac{dy}{dx} = 5x^4 - 18x^2 - 3$	M1 A1 A1	3	Decrease one power by 1 One term correct All correct
(b)(i)	Sub $x = 2$ into their $\frac{dy}{dx}$ Shown to equal 5	M1 A1	2	$80 - 72 - 3$ AG (be convinced)
(ii)	Gradient of normal $= -\frac{1}{5}(y + \frac{1}{5}x + \dots)$ $y - 3 = -\frac{1}{5}(x - 2)$ $x + 5y = 17$ (integer coefficients)	B1 M1 A1	3	Or $m_1 m_2 = -1$ used or stated Trying normal NOT tangent or $y = mx + c$ and attempt to find c Or integer multiple of coefficients
(c)	Sub $x = 1$ into their $\frac{dy}{dx}$ ($= -16 < 0$) Negative value \Rightarrow DECREASING	M1 E1 \checkmark	2	($5 - 18 - 3 = -16$) (Watch $\frac{d^2y}{dx^2} = -16$!) Correct interpretation of sign of $\frac{dy}{dx}$
Total			10	
3(a)	$(x - 6)^2 + (y - 3)^2$ $= 36 + 9 - 20$ $= 5^2$	B1 M1 A1	3	Generous with sign errors Condone 25
(b)	(i) Centre (6,3) (ii) Radius = 5	B1 \checkmark B1 \checkmark	2	fit their a and b Correct or fit \sqrt{RHS} if $RHS > 0$
(c)(i)	$x^2 + (x + 4)^2 - 12x - 6(x + 4) + 20 = 0$ $(2x^2 - 10x + 12 = 0) \Rightarrow x^2 - 5x + 6 = 0$	M1 A1	2	Or their $(x - a)^2 + (x + 4 - b)^2 = r^2$ AG (be convinced)
(ii)	$(x - 3)(x - 2) = 0$ $x = 2, x = 3$ P, Q are (2,6) and (3,7)	M1 A1 m1 A1	4	Attempt at factors or use of formula Both correct Substituting for one y value Both points correct
Total			11	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$f(-1) = -1 - 3 + 6 + 8$ (Remainder) = 10	M1	2	Or long division up to remainder term
		A1		
(ii)	$x - 1$ is a factor $x + 2$ is a factor	B1	2	May be earned retrospectively From part (iii)
		B1		
(iii)	Attempt at third factor $f(x) = (x - 1)(x + 2)(x - 4)$	M1	2	Multiplying/ dividing/factor theorem $(x+4) \Rightarrow$ M1, A0
		A1		
(b)(i)	At A , $y = 8$	B1	1	Or (0,8)
(ii)	At B , $x = 4$	B1	1	Or (4,0) NO ft of wrong factor
(c)(i)	$\frac{x^4}{4} - x^3 - 3x^2 + 8x$ (+c)	M1	4	Increase one power by 1 One term correct (unsimplified) Two other terms correct (unsimplified) All correct (unsimplified) (condone missing + c)
		A1		
		A1		
		A1		
(ii)	Realisation that limits are -2 and 1 Area = $\left[\frac{1}{4} - 1 - 3 + 8 \right] - [4 + 8 - 12 - 16]$ $= 20\frac{1}{4}$	B1	3	Condone wrong way round
		M1	3	Attempt to sub their limits into their (c)(i)
		A1		CSO. Must use $F(1) - F(-2)$ correctly
Total			15	
5(a)	$(\sqrt{12})^2 - 2^2$ attempt to multiply out (= $12 - 4$) = 8	M1	2	May have $\sqrt{12}$ terms
		A1		
(b)	$2\sqrt{3}$	B1	1	
(c)	Multiplying top and bottom by $\sqrt{12} + 2$ Numerator = $12 + 4\sqrt{12} + 4$ Expression = $\frac{16 + 4\sqrt{12}}{8}$ or $\frac{16 + 8\sqrt{3}}{8}$ $= 2 + \sqrt{3}$	B1	4	Or $\sqrt{3} + 1$ etc At least 3 terms multiplied out on top OE in $\sqrt{3}$ ft denominator from (a); or correct but numerator correct (unsimplified)
		M1		
		A1✓	4	
		A1		
Total			7	

MPC1 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Sides $24 - 2x$, $9 - 2x$ $V = x(24 - 2x)(9 - 2x)$ $= 4x^3 - 66x^2 + 216x$	B1 M1 A1	3	Either correct 3 sides involving x multiplied together AG (be convinced)
(b)(i)	$\frac{dV}{dx} = 12x^2 - 132x + 216$	M1 A1 A1	3	Power decreased by 1 One term correct All correct (no $+C$ etc)
(ii)	Putting their $\frac{dV}{dx} = 0$ (must see this first) $\Rightarrow x^2 - 11x + 18 = 0$	M1 A1	2	Or their $12x^2 - 132x + 216 = 0$ Or $12(x^2 - 11x + 18) = 0$ or statement AG (be convinced)
(iii)	$(x - 2)(x - 9) = 0$ $\Rightarrow x = 2, x = 9$	M1 A1	2	Factors, comp sq or formulae used (1 slip)
(iv)	Reject $x = 9$, since $9 - 2x < 0$	E1	1	$x = 2$ is only possible value
(c)(i)	$\frac{d^2V}{dx^2} = 24x - 132$	M1 A1	2	Differentiating their $\frac{dV}{dx}$ (eg $2x - 11$) Correct
(ii)	$x = 2$ only $\Rightarrow \frac{d^2V}{dx^2} = -84$ (or < 0) Maximum value	B1 E1✓	2	Correct $\frac{d^2V}{dx^2}$ value OE full test. ft if their test implies minimum
Total			15	
7(a)	$k^2 + 10k + 25 - 12k^2 - 24k$ $= -11k^2 - 14k + 25$	M1 A1	2	Condone one slip No ISW here
(b)(i)	Real roots when " $b^2 - 4ac$ " ≥ 0 $(k + 5)^2 - 12k(k + 2)$ $(k - 1)(11k + 25)$ attempted to be shown equal to $11k^2 + 14k - 25$ $-11k^2 - 14k + 25 \geq 0$ $\Rightarrow (k - 1)(11k + 25) \leq 0$	B1 M1 m1 A1 A1	5	Non-negative discriminant (stated / used) Finding $b^2 - 4ac$ in terms of k Or factorisation attempt Real roots condition correct and ... AG (be convinced about inequality)
(ii)	(Critical values) 1 and $-\frac{25}{11}$ seen Sketch or sign diagram $\Rightarrow -\frac{25}{11} \leq k \leq 1$	B1 M1 A1	3	$\begin{array}{c} + \qquad \qquad \qquad + \\ \hline \qquad \qquad \qquad \qquad \qquad \qquad \\ \qquad \qquad \qquad - \frac{25}{11} \qquad \qquad \qquad 1 \end{array}$
Total			10	
TOTAL			75	